AD-A153 803

NSWC TR 83-135

PROGRAM KMEC: THE COMPUTATION OF KOZAI MEAN ORBITAL ELEMENTS USING A NONSINGULAR FORMULATION

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SEPTEMBER 1983

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2

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SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE

READ INSTRUCTIONS BEFORE COMPLETING FORM

1. REPORT NUMBER	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
NSWC TR 83-135	AD-A153 803	

4. TITLE (and Subtitle)	5. TYPE OF REPORT & PERIOD COVERED
PROGRAM KMEC: THE COMPUTATION OF KOZAI MEAN ORBITAL ELEMENTS USING A NONSINGULAR FORMULATION	Final
	6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s)	8. CONTRACT OR GRANT NUMBER(s)
A. D. Parks	
9. PERFORMING ORGANIZATION NAME AND ADDRESS	10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
Naval Surface Weapons Center (K13) Dahlgren, VA 22448	35159B; 3K12CS
11. CONTROLLING OFFICE NAME AND ADDRESS	12. REPORT DATE
Defense Mapping Agency Washington, DC 20305	September 1983
	13. NUMBER OF PAGES
	58
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)	15. SECURITY CLASS. (of this report)
	UNCLASSIFIED
	15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

DTIC
ELECTE
MAY 16 1985
S D
A

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Kozai mean element converter (KMEC)
Orbital element sets
MX 1502-DS geoceiver
Modified TRANET II geoceiver

orbital element sets for satellites tracked by

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

A geopotential expressed in terms of a nonsingular set of orbital elements is used to iteratively compute Kozai mean orbital elements from either Brouwer mean orbital elements or inertial osculating Cartesian position and velocity vectors. Although the computational procedure is general in nature, KMEC has been tailored to generate Kozai mean elements ~~in a format that is directly applicable for use in~~ the MX 1502-DS and modified TRANET II geoceivers. Additional keywords: KMEC computer program; KMEC (Kozai mean element converter); Kalman filtering.

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S/N 0102-LF-014-6601

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FOREWORD

This document provides a detailed description of the formulation and computational processes contained within the Kozai Mean Element Converter (KMEC) software package. The KMEC program was developed by the Naval Surface Weapons Center under the auspices of the Defense Mapping Agency to specifically provide operational support to the new MX 1502-DS and modified TRANET II Doppler beacon satellite tracking receivers. This report has been reviewed and approved by Dr. R. J. Anderle.

Released by:

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INTRODUCTION

The primary function of the Kozai mean element converter (KMEC) is to generate mean Kozai orbital element sets for satellites tracked by the MX 1502-DS and modified TRANET II geoceivers. These geoceivers will use the mean element sets in conjunction with a Kalman filter to provide autonomous orbit updates and to predict station-satellite inview periods. To avoid problems associated with mathematical singularities, such as those that occur with near-circular orbits, the following nonsingular element set has been selected to perform the mean element transformation in KMEC:

$$\left.\begin{aligned} a &= \text{semimajor axis} \\ \lambda &= \ell + g + h \\ \xi &= e \cos \tilde{\omega} \qquad (\tilde{\omega} = g + h) \\ \eta &= e \sin \tilde{\omega} \\ P &= \sin\left(\frac{i}{2}\right) \cos h \\ Q &= \sin\left(\frac{i}{2}\right) \sin h \end{aligned}\right\} \tag{1}$$

where a, e, i, ℓ, g, and h are the usual Keplerian elements.

KMEC is comprised of seven basic computational functions:

1. The process flow supervisor (PFS)
2. The Cartesian input section (CIS)
3. The Brouwer input section (BIS)
4. The Walter mean element interator (WMI)
5. The nonsingular orbital element builder (NEB)
6. The geoceiver format section (GFS)
7. The Keplerian element builder (KEB)

Figure 1 shows the functional overview of KMEC and the interfunctional data flow. A detailed description of each of these functions is presented in the following sections.

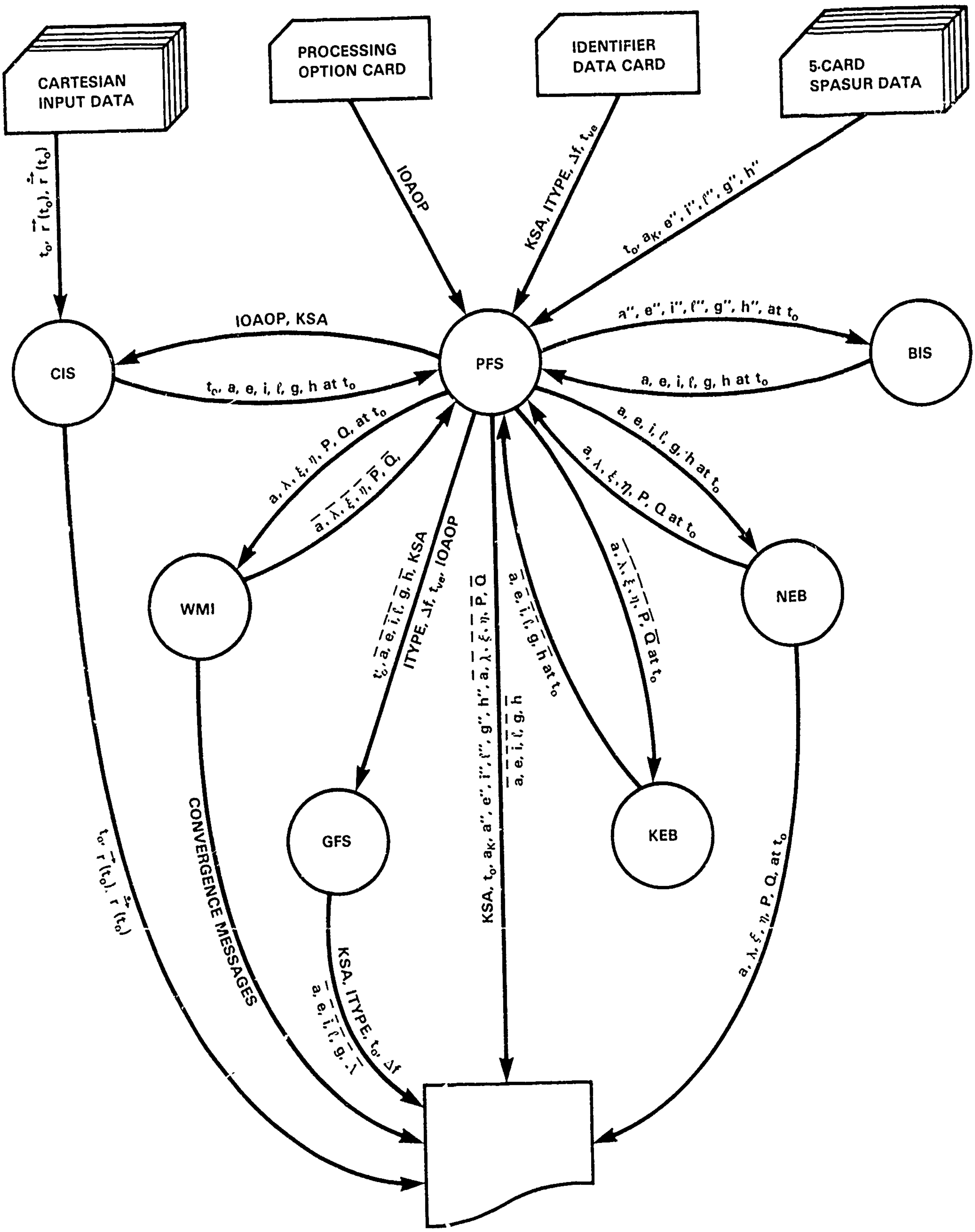

FIGURE 1. KMEC FUNCTIONAL OVERVIEW AND DATA FLOW

As user-supplied input, KMEC requires the selection of a processing option (IOAOP). If IOAOP = 0, KMEC will use a nonsingular transformation theory to convert a Brouwer mean element set obtained from Space Surveillance (SPASUR) input data to a Kozai mean element set. If IOAOP = 1, KMEC will use the same nonsingular transformation theory to convert osculating Cartesian position and velocity vectors to an associated Kozai mean element set. Also required as input are the satellite number (KSA), the satellite type (ITYPE), the satellite frequency offset (Δf), and the universal time of transit of the vernal equinox (t_{VE}). These data are included on the identifier data card. The Kozai mean elements and associated information are written to hard copy during the computational cycle.

THE PROCESS FLOW SUPERVISOR (PFS)

FUNCTIONAL DESCRIPTION

The principal tests performed by the PFS are to receive input data, direct the processing flow, and output computed results. Specifically, the PFS:

1. Receives from input the user-selected processing option
2. Receives from input the identifier card data
3. Receives from input Brouwer mean elements from five-card SPASUR data when IOAOP = 0
4. Directs processing through the CIS, BIS, WMI, NEB, KEB, and GFS functions
5. Converts input data to the proper computational units
6. Writes to hard copy the input, intermediate, and output mean element sets

The flow of the PFS function is presented in Figure 2.

When IOAOP = 1, the CIS function is entered and osculating Cartesian position and velocity vectors are received along with the vector epoch. These vectors are converted to an osculating Keplerian element set and are used to initiate the Kozai transformation process.

PROCESSING EQUATIONS

The semimajor axis read from the SPASUR data is the Kaula semimajor axis a_k expressed in earth radii. This is converted in the PFS to the Brouwer mean semimajor axis a'' via the transformation

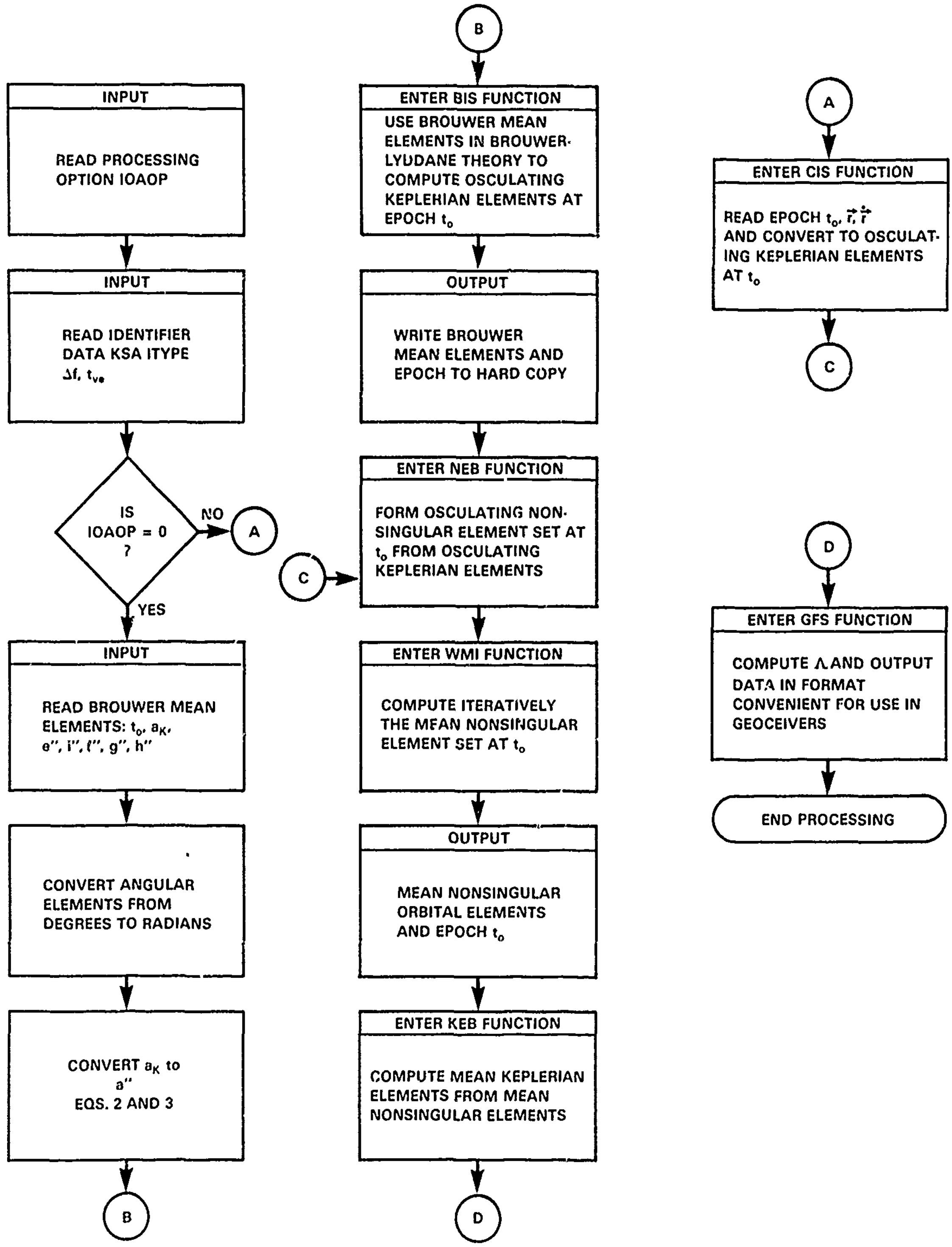

FIGURE 2. PROCESS FLOW SUPERVISOR LOGIC FLOW

$$a'' = a_k \; a_e \left(\frac{1 + 2X}{1 - X} \right)^{2/3} \tag{2}$$

where

$$X = \frac{3\,J_2(1 - 3/2\,\sin^2 i'')}{4\,a_k^2\,(1-e''^2)^{3/2}} \tag{3}$$

In the above expressions, a_e is the earth's semimajor axis, J_2 is a zonal harmonic gravitational constant, and i'' and e'' are the Brouwer mean inclination and eccentricity, respectively.

BROUWER INPUT SECTION (BIS)

FUNCTIONAL DESCRIPTION

The Brouwer input section accepts the Brouwer mean element set from the PFS and converts it into an associated osculating element set at epoch t_o. This is accomplished through the application of the Brouwer–Lyddane theory,[1,2] which has been modified to include the effects of atmospheric drag (the atmospheric drag decay rates are nulled during this computation). These osculating elements are then used to formulate the osculating nonsingular element set that initializes the mean element interation algorithm. The BIS processing logic flow is shown in Figure 3.

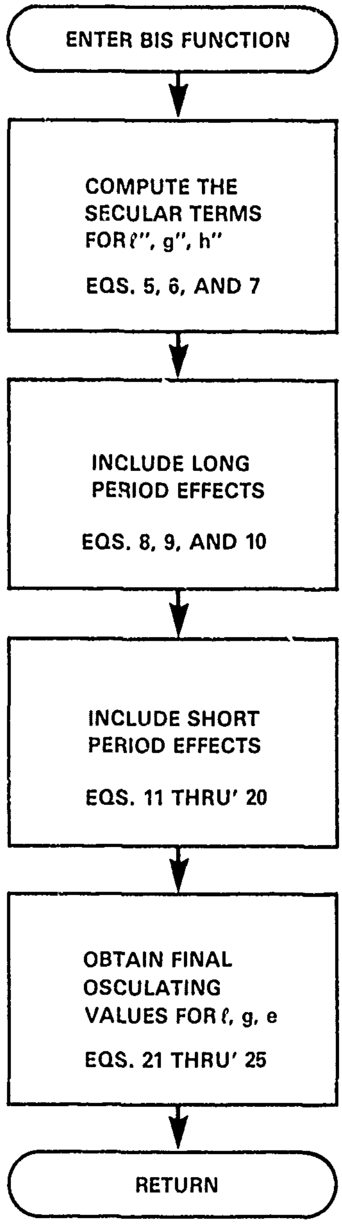

FIGURE 3. BROUWER INPUT SECTION PROCESS FLOW

PROCESSING EQUATIONS FOR THE BROUWER–LYDDANE METHOD

The equations used to compute osculating orbital elements from mean Brouwer elements and associated decay rates are delineated in this section. First define the following:

$$\left.\begin{array}{lcl}
\dot{a}'' & = & \text{semimajor axis decay rate} \\
\dot{e}'' & = & \text{eccentricity decay rate} \\
\dot{n} & = & \text{time rate of change of mean motion} \\
t & = & \text{time from epoch} \\
n_o & = & (\mu/a''^3)^{1/2} \\
\eta & = & (1 - e''^2)^{1/2} \\
\theta & = & \cos i'' \\
\gamma_2 & = & 1/2\ C_{20}\ a_e^2/a''^2 \\
\gamma_2' & = & \gamma_2\eta^{-4} \\
\gamma_3' & = & -C_{30}\ a_e^3\ a''^{-3}\eta^{-6} \\
\gamma_4' & = & -3/8\ C_{40}\ a_e^4\ a''^{-4}\eta^{-8} \\
\gamma_5' & = & -C_{50}\ a_e^5\ a''^{-5}\eta^{-10} \\
\alpha & = & 1 - 5\theta^2 \\
\beta & = & 1 - 11\theta^2 - 40\theta^4\ \alpha^{-1} \\
\gamma & = & 1 - 3\theta^2 - 8\theta^4\ \alpha^{-1} \\
\delta & = & 1 - 9\theta^2 - 24\theta^4\ \alpha^{-1} \\
\lambda & = & 1 - 5\theta^2 - 16\theta^4\ \alpha^{-1}
\end{array}\right\} \qquad (4)$$

where the C_{j0} (j = 2, 3, 4, 5) are the zonal harmonic gravitational expansion coefficients. Then the secular terms are computed from

$$\ell'' = n_0 t \left\{ 1 + \frac{3}{2}\gamma_2'\eta(3\theta^2 - 1) + \frac{3}{32}\gamma_2'^2\eta \left[-15 + 16\eta + 25\eta^2 + (30 - 96\eta - 90\eta^2)\theta^2 + (105 + 144\eta + 25\eta^2)\theta^4 \right] + \frac{15}{16}\gamma_4'\ \eta e''^2 \left[3 - 30\theta^2 + 35\theta^4 \right] \right\} + \ell_0'' + \dot{n} t^2 \qquad (5)$$

$$g'' = n_0 t \left\{ - \frac{3}{2} \gamma_2' \alpha + \frac{3}{32} \gamma_2'^2 \left[-35 + 24\eta + 25\eta^2 \right. \right.$$

$$\left. + (90 - 192\eta - 126\eta^2)\theta^2 + (385 + 360\eta + 45\eta^2)\theta^4 \right] \quad (6)$$

$$\left. + \frac{5}{16} \gamma_4' \left[21 - 9\eta^2 + (-270 \pm 126\eta^2)\theta^2 + (385 - 189\eta^2)\theta^4 \right] \right\} + g_0''$$

and

$$h'' = n_0 t \left\{ - 3\gamma_2'\theta + \frac{3}{8} \gamma_2'^2 \left[(-5 + 12\eta + 9\eta^2)\theta + (-35 - 36\eta - 5\eta^2)\theta^3 \right] \right.$$

$$\left. + \frac{5}{4} \gamma_4' (5 - 3\eta^2)\theta (3 - 7\theta^2) \right\} + h_0'' \quad (7)$$

The long period (dependent upon g″) terms are computed from

$$\delta_1 e = \frac{35}{96} \frac{\gamma_5'}{\gamma_2'} e''^2 \eta^2 \lambda \sin i'' \sin^3 g'' - \frac{1}{12} \frac{e''\eta^2}{\gamma_2'} (3\gamma_2'^2 \beta - 10\gamma_4' \gamma) \sin^2 g''$$

$$- \frac{35}{128} \frac{\gamma_5'}{\gamma_2'} e''^2 \eta^2 \lambda \sin i'' \sin g'' + \frac{1}{4} \frac{\eta^2}{\gamma_2'} \left[\gamma_3' + \frac{5}{16} \gamma_5' (4 + 3e''^2) \delta \right]$$

$$\sin i'' \sin g'' + \frac{e''\eta^2}{24\gamma_2'} \left[3\gamma_2'^2 \beta - 10\gamma_4' \gamma \right] \quad (8)$$

$$\ell' + g' = g'' + \ell'' + \tfrac{1}{2} \left\{ \frac{1}{24\gamma_2'} \left[- 3\gamma_2'^2 \left\{ 2 + e''^2 - 11(2 + 3e''^2)\theta^2 \right. \right. \right.$$

$$\left. - 40(2 + 5e''^2)\theta^4 \alpha^{-1} - 400e''^2 \theta^6 \alpha^{-2} \right\}$$

$$\left. + 10\gamma_4 \left\{ 2 + e''^2 - 3(2 + 3e''^2)\theta^2 - 8(2 + 5e''^2)\theta^4 \alpha^{-1} - 80e''^2 \theta^6 \alpha^{-2} \right\} \right]$$

$$+ \frac{\eta^3}{\gamma_2'} \left[\frac{\gamma_2'^2}{4} \quad \beta - \frac{5}{6} \gamma_4' \gamma \right] \Bigg\} \sin\ 2g'' + \Bigg\{ \frac{35}{384} \frac{\gamma_5'}{\gamma_2'} \eta^3 e'' \ \lambda \ \sin\ i''$$

$$+ \frac{35}{1152} \frac{\gamma_5'}{\gamma_2'} \left[\lambda \left\{ - e''(3 + 2e''^2) \sin\ i'' + \frac{e''^3 \theta^2}{\sin\ i''} \right\} \right.$$

$$\left. + 2e''^3 \theta^2 \ \sin\ i'' \left\{ 5 + 32\theta^2 \ \alpha^{-1} + 80\theta^4 \ \alpha^{-2} \right] \right] \Bigg\} \cos\ 3g'' \tag{9}$$

$$+ \Bigg\{ - \frac{\gamma_3 e'' \theta^2}{4\gamma_2' \sin i''} + \frac{5}{64} \frac{\gamma_5'}{\gamma_2'} \left[- e'' \frac{\theta^2}{\sin i''} (4 + 3e''^2) + e'' \sin i'' \right.$$

$$\left. (26 + 9e''^2) \right] \delta - \frac{15}{32} \frac{\gamma_5'}{\gamma_2'} \ e'' \theta^2 \ \sin i'' \ (4 + 3e''^2)$$

$$(3 + 16\theta^2 \alpha^{-1} + 40\theta^4 \alpha^{-2}) + ¼ \frac{\gamma_3'}{\gamma_2'} \ \sin i''.$$

$$\left(\frac{e''}{1 + \eta^3} \right) \left[3 - e''^2 \ (3 - e''^2) \right] + \frac{5}{64} \frac{\gamma_5'}{\gamma_2'} \ \eta^2 \delta$$

$$\left[\frac{e''(- 32 + 81e''^4)}{4 + 3e''^2 + \eta(4 + 9e''^2)} \right] \ \sin i'' \Bigg\} \cos g''$$

and

$$h' = h'' + \frac{35\gamma_5' e''^3 \theta}{144\gamma_2'} \ \left\{ \frac{\lambda}{2} \sin^{-1}\ i'' + \sin i'' \left[5 + 32\theta^2 \alpha^{-1} + 80\theta^4 \alpha^{-2} \right] \right\}$$

$$\sin^2\ g''\ \cos\ g'' + \frac{e''^2 \theta}{12\gamma_2'} \Bigg\{ - 3\gamma_2'^2 \left[11 + 80\theta^2 \alpha^{-1} + 200\theta^4 \alpha^{-2} \right] \tag{10}$$

$$+ 10\gamma_4' \left[3 + 16\theta^2 \alpha^{-1} + 40\theta^4 \alpha^{-2} \right] \Bigg\} \sin\ g''\ \cos\ g''$$

$$+ \Bigg\{ - \frac{35\gamma_5'}{576\gamma_2'} \ e''^3 \theta \left[\frac{\lambda}{2} \sin^{-1}\ i'' + \sin\ i''\ (5 + 32\theta^2 \alpha^{-1} \ 80\theta^4 \alpha^{-2}) \right]$$

$$+ \frac{e'' \theta}{4\gamma_2' \ \sin\ i''} \ \left[\gamma_3' + \frac{5}{16} \gamma_5' \ (4 + 3e''^2) \delta + \frac{15}{8} \gamma_5' \ (4 + 3e''^2) \right.$$

$$\left. (3 + 160\theta^2 \alpha^{-1} + 40\theta^4 \alpha^{-2}) \ \sin^2 i'' \right] \Bigg\} \cos\ g'' \ .$$

The short periodics (dependent upon E', f', ℓ'') are computed from:

$$a = \dot{a}''t + a'' - a''\frac{\gamma_2}{\eta^3}(3\theta^2 - 1) + \left[\frac{a''\gamma_2}{(1 - e''\cos E')^3}\right] \tag{11}$$

$$[3\theta^2 - 1 + 3\sin^2 i''\cos(2g'' + 2f')]$$

$$e = e'' + \dot{e}''t + \delta_1 e + \frac{\eta^2\gamma_2}{2}\left\{\frac{3\theta^2 - 1}{\eta^6}\left[\frac{e''}{1 + \eta^3}\left\{3 - e''^2(3 - e''^2)\right\}\right.\right. \tag{12}$$

$$\left.\left. + \left\{3 + e''\cos f' \cdot (3 + e''\cos f')\right\}\cos f'\right] + \frac{3(1 - \theta^2)}{\eta^6}\right.$$

$$\left.\left[e'' + \left\{3 + e''\cos f'(3 + e''\cos f')\right\}\cos f'\right]\cos(2f' + 2g'')\right\}$$

$$- \frac{\eta^2\gamma_2'}{2}(1 - \theta^2)\left[3\cos(2g'' + f') + \cos(2g'' + 3f')\right]$$

$$i = i'' - \frac{e''\theta}{\eta^2\sin i''}\delta_1 e + e''\gamma_2'\theta\sin i''\sin f'\sin(2f' + 2g'') \tag{13}$$

$$+ 2e''\gamma_2'\theta\sin i'' \cdot \cos f'\cos(2f' + 2g'') + \frac{3}{2}\gamma_2'\theta\sin i''\cos(2f' + 2g'')$$

$$g + \ell = g' + \ell' + \frac{\gamma_2'}{4}\left\{- 6\alpha(f' - \ell'' + e''\sin f') + (3 - 5\theta^2)\right.$$

$$\left.\left[3\sin(2f' + 2g'') + 3e''\sin(2g'' + f') + e''\sin(2g'' + 3f')\right]\right\} \tag{14}$$

$$+ \frac{e''\eta^2\gamma_2'}{4(1 + \eta)}\left\{2(3\theta^2 - 1)(\sigma + 1)\sin f' + 3(1 - \theta^2)\right.$$

$$\left.\left[(1 - \sigma)\sin(2g'' + f') + (\sigma + 1/3)\sin(2g'' + 3f')\right]\right\}$$

$$h = h' + \left[2e''\gamma_2'\theta\cos f' + \frac{3}{2}\gamma_2'\theta\right]\sin(2g'' + 2f') \tag{15}$$

$$- e''\gamma_2'\theta\sin f'\cos(2f' + 2g'') - 3\gamma_2'\theta(f' - \ell'' + e''\sin f')$$

and

$$e\delta\ell = \frac{1}{2}\,\frac{e''\eta^3}{\gamma_2'}\left\{\frac{1}{4}\,\gamma_2'\beta - \frac{5}{6}\,\gamma_4'\gamma\right\}\sin\,2g''$$

$$-\left\{\frac{1}{4}\,\frac{\gamma_3'}{\gamma_2'}\,\eta^3\,\sin\,i'' + \frac{5}{64}\,\frac{\gamma_5'}{\gamma_2'}\,\eta^3\,\sin\,i''\,(4 + 9e^{2\,''})\delta\right\}\,. \tag{16}$$

$$\cos\,g'' + \frac{35}{384}\,\frac{\gamma_5'}{\gamma_2'}\,\eta^3 e''\,\lambda\,\sin\,i''\,\cos\,3g''$$

$$-\,\frac{1}{4}\,\gamma_2'\eta^3\left\{2(3\theta^2 - 1)\,(\sigma + 1)\,\sin\,f' + 3(1 - \theta^2)\,(1 - \sigma)\,\sin\,(2g'' + f')\right.$$

$$\left.+\,\left(\sigma + \frac{1}{3}\right)\,\sin\,(2g'' + 3f')\right\}$$

where

$$\sigma = \left(\frac{\eta}{1 - e''\,\cos\,E'}\right)^2 + \left(\frac{1}{1 - e''\,\cos\,E'}\right) \tag{17}$$

The eccentric anomaly E' is obtained from a Newton–Raphson iteration upon the Kepler equation

$$E' - e''\,\sin\,E' = \ell'' \tag{18}$$

and the true anomaly f' is found from

$$\sin\,f' = \frac{\eta\,\sin\,E'}{1 - e''\,\cos\,E'} \tag{19}$$

$$\cos\,f' = \frac{\cos\,E' - e''}{1 - e''\,\cos\,E'} \tag{20}$$

The final osculating values for a, i, and h are computed from Equations 11, 13, and 15, respectively. Equations 5, 12, 14, and 16 are used to calculate final osculating values for ℓ, g, and e for the following relations:

$$A = e \cos \ell'' - e\delta\ell \sin \ell'' \tag{21}$$

$$B = e \sin \ell'' + e\delta\ell \cos \ell'' \tag{22}$$

$$\ell = \tan^{-1} (B/A) \tag{23}$$

$$g = (\ell + g) - \ell \tag{24}$$

and

$$e = (A^2 + B^2)^{1/2} \tag{25}$$

CARTESIAN INPUT SECTION (CIS)

FUNCTIONAL DESCRIPTION

The primary tasks performed by the CIS function are to

1. Receive from input inertial Cartesian position and velocity vectors at epoch t_o, i.e., $\vec{r}(t_o)$ and $\dot{\vec{r}}(t_o)$
2. Transform the osculating inertial Cartesian components to osculating Keplerian orbital elements

The process flow of the CIS function is shown in Figure 4.

PROCESSING EQUATIONS

The osculating inertial Cartesian vectors $\vec{r}(t_o) = (x, y, z)$ and $\dot{\vec{r}}(t_o) = (\dot{x}, \dot{y}, \dot{z})$ are transformed to osculating Keplerian orbital elements by using the following relationships.

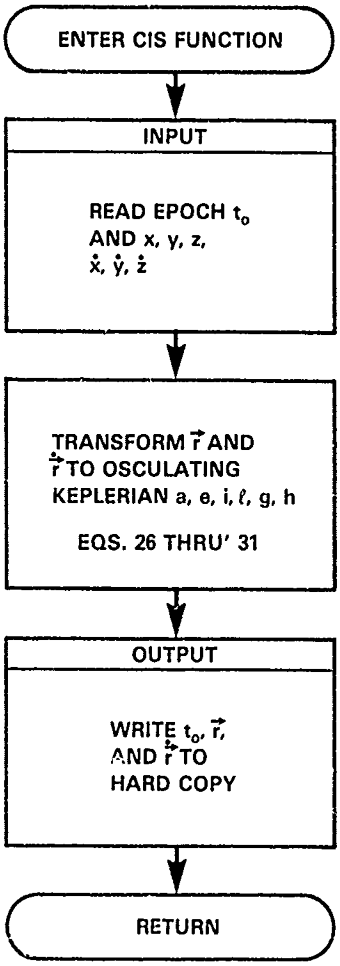

FIGURE 4. CARTESIAN INPUT SECTION PROCESS FLOW

$$a = \left[\frac{2}{|\vec{r}|} - \frac{|\dot{\vec{r}}|^2}{\mu}\right]^{-1} \tag{26}$$

$$e = \left\{\frac{(\vec{r} \cdot \dot{\vec{r}})^2}{\mu a} + \left[\frac{|\vec{r}|\ |\dot{\vec{r}}|^2}{\mu} - 1\right]^2\right\}^{1/2} \tag{27}$$

$$i = \tan^{-1}\left\{\frac{\left[(y\dot{z} - z\dot{y})^2 + (z\dot{x} - x\dot{z})^2\right]^{1/2}}{x\dot{y} - y\dot{x}}\right\} \tag{28}$$

$$\ell = \tan^{-1}\left\{\frac{(x\dot{y} - y\dot{x})\ y}{x|x\dot{y} - y\dot{x}|}\right\} \tag{29}$$

$$h = \tan^{-1}\left\{\frac{y\dot{z} - z\dot{y}}{x\dot{z} - z\dot{x}}\right\} \tag{30}$$

and

$$g = \tan^{-1}\left\{\frac{z\ |\vec{r} \times \dot{\vec{r}}|\left[\frac{|\vec{r}|\ |\dot{\vec{r}}|^2}{\mu} - 1 - e^2\right] + \left[x(z\dot{x} - x\dot{z}) - y(y\dot{z} - z\dot{y})\right]\sqrt{1 - e^2}\ \frac{\vec{r} \cdot \dot{\vec{r}}}{(\mu a)^{1/2}}}{z|\vec{r} \times \dot{\vec{r}}|\sqrt{1 - e^2}\ \frac{\vec{r} \cdot \dot{\vec{r}}}{(\mu a)^{1/2}} - \left[x(z\dot{x} - x\dot{z}) - y(y\dot{z} - z\dot{y})\right]\left[\frac{|\vec{r}|\ |\dot{\vec{r}}|^2}{\mu} - 1 - e^2\right]}\right\} \tag{31}$$

THE NONSINGULAR ORBITAL ELEMENT BUILDER (NEB)

The NEB function uses the osculating Keplerian elements obtained from either the BIS or CIS functions to form the osculating nonsingular element set given by Equation 1. This nonsingular element set is used to initialize the Walter mean element iterator discussed in the following section. The NEB process flow is shown in Figure 5.

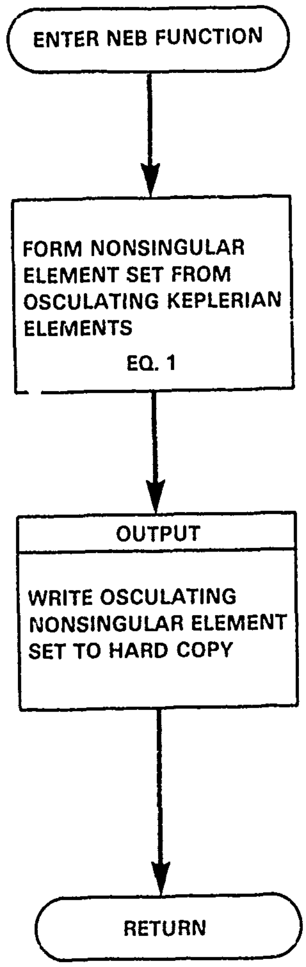

FIGURE 5. THE NEB FUNCTION PROCESS FLOW

THE WALTER MEAN ELEMENT ITERATOR FUNCTION (WMI)

FUNCTIONAL DESCRIPTION

The primary tasks performed by the WMI are to

1. Iteratively solve for the mean nonsingular element set associated with the osculating nonsingular element set formed by the NEB function

2. Write to hard copy a message describing the convergence status of the WMI algorithm

The mathematical computations performed by this function are relatively lengthy and quite complex. They are described in detail in the next subsection. The WMI process flow is depicted in Figure 6.

PROCESSING EQUATIONS

The iterative technique used to find the mean nonsingular element set from the associated osculating elements is similar to that described by Walter.[3] This mean nonsingular element set, represented by $\bar{\beta}_j$, where

$$\bar{\beta}_j = \begin{cases} \bar{a} & , j = 1 \\ \bar{\lambda} & , j = 2 \\ \bar{\xi} & , j = 3 \\ \bar{\eta} & , j = 4 \\ \bar{P} & , j = 5 \\ \bar{Q} & , j = 6 \end{cases} \qquad (32)$$

is obtained from the iterative process executed according to the scheme

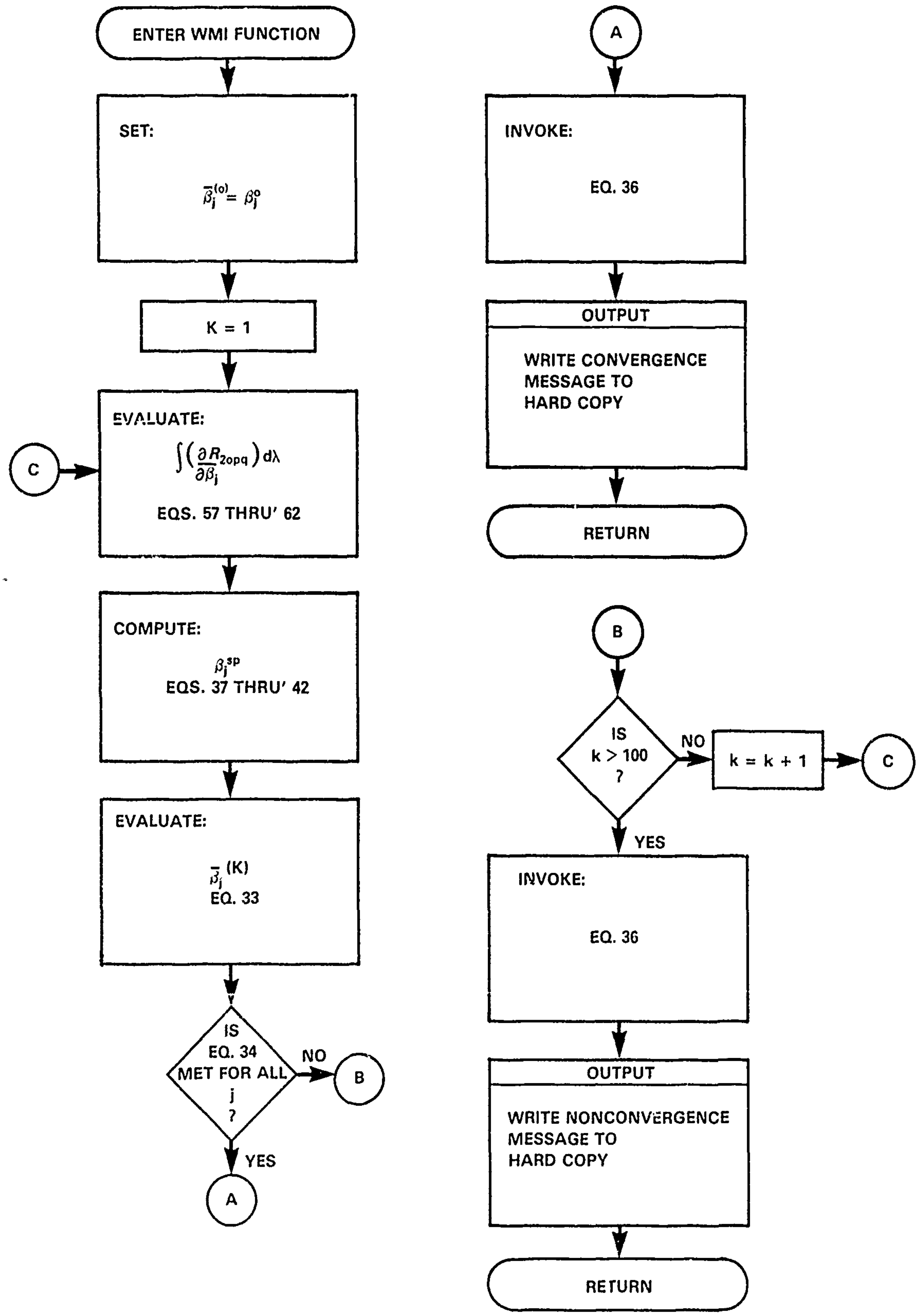

FIGURE 6. THE WMI FUNCTION PROCESS FLOW

$$\overline{\beta}_j^{(k)}(t_o) = \beta_j^o(t_o) - \beta_j^{SP}\left(\overline{\beta}_1^{(k-1)}(t_o), \ldots, \overline{\beta}_6^{(k-1)}(t_o)\right), \quad (j = 1, 2, \ldots, 6), \tag{33}$$

until the condition

$$\left|\overline{\beta}_j^{(k)}(t_o) - \overline{\beta}_j^{(k-1)}(t_o)\right| < \epsilon_j, \quad (j = 1, 2, \ldots, 6) \tag{34}$$

is satisfied for all j. In the above expressions, k is the iteration counter; $\beta_j^o(t_o)$ and $\beta_j^{SP}(t_o)$ represent the osculating values and the short periodic variation of the j^{th} element at epoch t_o; and ϵ_j is the convergence tolerance for the j^{th} element.

To initiate this iterative process, it is assumed that

$$\overline{\beta}_j^{(o)}(t_o) = \beta_j^o(t_o), \quad (j = 1, 2, \ldots, 6). \tag{35}$$

When the condition in Equation 34 is met, then

$$\overline{\beta}_j(t_o) = \overline{\beta}_j^{(k)}(t_o), \quad (j = 1, 2, \ldots, 6) \tag{36}$$

As can be seen from Equation 33, short periodic variations for the nonsingular element set are required. These can be obtained by integrating the associated Lagrange planetary equations using only the J_2 zonal harmonic in the gravitational disturbing function:

$$a^{SP} = J_2\left(\frac{a_e^2}{a}\right)\left\{\left(\frac{a}{r}\right)^3\left[1 - \frac{3}{2}\sin^2 i + \frac{3}{2}\sin^2 i \cos 2(\omega + f)\right] - \left(1 - \frac{3}{2}\sin^2 i\right)(1 - e^2)^{-3/2}\right\} \tag{37}$$

$$\lambda^{SP} = -\frac{2}{n^2 a} \sum_{pq} \int \left(\frac{\partial R_{20pq}}{\partial a} \right) d\lambda + \frac{\gamma}{2n^2 a^2} \left[\xi \sum_{pq} \int \left(\frac{\partial R_{20pq}}{\partial \xi} \right) d\lambda \right.$$

$$\left. + \eta \sum_{pq} \int \left(\frac{\partial R_{20pq}}{\partial \eta} \right) d\lambda \right] + \frac{1}{2n^2 a^2 \gamma} \left[P \sum_{pq} \int \left(\frac{\partial R_{20pq}}{\partial P} \right) d\lambda \right. \tag{38}$$

$$\left. + Q \sum_{pq} \int \left(\frac{\partial R_{20pq}}{\partial Q} \right) d\lambda \right]$$

$$\xi^{SP} = -\frac{\gamma}{n^2 a^2 (1+\gamma)} \xi \sum_{pq} \int \left(\frac{\partial R_{20pq}}{\partial \lambda} \right) d\lambda - \frac{\gamma}{n^2 a^2} \sum_{pq} \int \left(\frac{\partial R_{20pq}}{\partial \eta} \right) d\lambda$$

$$- \frac{1}{2n^2 a^2 \gamma} \eta \left[P \sum_{pq} \int \left(\frac{\partial R_{20pq}}{\partial P} \right) d\lambda + Q \sum_{pq} \int \left(\frac{\partial R_{20pq}}{\partial Q} \right) d\lambda \right] \tag{39}$$

$$\eta^{SP} = -\frac{\gamma}{n^2 a^2 (1+\gamma)} \eta \sum_{pq} \int \left(\frac{\partial R_{20pq}}{\partial \lambda} \right) d\lambda + \frac{\gamma}{n^2 a^2} \sum_{pq} \int \left(\frac{\partial R_{20pq}}{\partial \xi} \right) d\lambda$$

$$+ \frac{1}{2n^2 a^2 \gamma} \xi \left[P \sum_{pq} \int \left(\frac{\partial R_{20pq}}{\partial P} \right) d\lambda + Q \sum_{pq} \int \left(\frac{\partial R_{20pq}}{\partial Q} \right) d\lambda \right] \tag{40}$$

$$P^{SP} = -\frac{1}{2n^2 a^2 \gamma} P \sum_{pq} \int \left(\frac{\partial R_{20pq}}{\partial \lambda} \right) d\lambda - \frac{1}{4n^2 a^2 \gamma} \sum_{pq} \int \left(\frac{\partial R_{20pq}}{\partial Q} \right) d\lambda$$

$$+ \frac{1}{2n^2 a^2 \gamma} P \left[\eta \sum_{pq} \int \left(\frac{\partial R_{20pq}}{\partial \xi} \right) d\lambda - \xi \sum_{pq} \int \left(\frac{\partial R_{20pq}}{\partial \eta} \right) d\lambda \right] \tag{41}$$

and

$$Q^{SP} = -\frac{1}{2n^2a^2\gamma} Q \sum_{pq} \int \left(\frac{\partial R_{20pq}}{\partial \lambda}\right) d\lambda + \frac{1}{4n^2a^2\gamma} \sum_{pq} \int \left(\frac{\partial R_{20pq}}{\partial P}\right) d\lambda$$

$$+ \frac{1}{2n^2a^2\gamma} Q \left[\eta \sum_{pq} \int \left(\frac{\partial R_{20pq}}{\partial \xi}\right) d\lambda - \xi \sum_{pq} \int \left(\frac{\partial R_{20pq}}{\partial \eta}\right) d\lambda\right] \tag{42}$$

In the above expressions, f is the true anomaly

$$\gamma = \sqrt{1 - e^2} \tag{43}$$

$$r = \frac{a(1 - e^2)}{1 + e \cos f} \tag{44}$$

and R_{20pq} is the $(pq)^{th}$ contribution to the geopotential disturbing function due to the J_2 zonal harmonic and is given in general by:[4]

$$R_{\ell mpq} = \frac{\mu a_e^\ell}{a^{\ell+1}} J^{(c)}_{\ell mp} K^{(\gamma)}_{\ell pq} \Big\{ \mathcal{R}_{\ell mpq} (A_{\ell m} \cos \theta_{\ell mpq} + B_{\ell m} \sin \theta_{\ell mpq}) +$$

$$\mathrm{II}_{\ell mpq}(A_{\ell m} \sin \theta_{\ell mpq} - B_{\ell m} \cos \theta_{\ell mpq}) \Big\} \tag{45}$$

where

$$A_{\ell m} = \begin{cases} C_{\ell m}, & \ell\text{-m even} \\ -S_{\ell m}, & \ell\text{-m odd} \end{cases} \tag{46}$$

$$B_{\ell m} = \begin{cases} S_{\ell m}, & \ell\text{-m even} \\ C_{\ell m}, & \ell\text{-m odd} \end{cases} \tag{47}$$

and

$$\theta_{\ell mpq} = (\ell - 2p + q)\lambda - m\theta \tag{48}$$

Here θ is the Greenwich sidereal time. Note that for the case $\ell = 2$, $m = 0$

$$A_{20} = C_{20} = -J_2 \tag{49}$$

$$B_{20} = S_{20} = 0 \tag{50}$$

and

$$\theta_{20pq} = (2 - 2p + q)\lambda \tag{51}$$

The $J_{\ell mp}$ and $K_{\ell pq}$ functions in Equation 44 are the inclination and eccentricity functions given by

$$J^{(c)}_{\ell mp} = (-1)^k \frac{(\ell+m)!}{2^\ell p!(\ell-p)!} \sum_{j=j_1}^{j2} (-1)^j \binom{2\ell-2p}{j}\binom{2p}{\ell-m-j} C^{2\ell-\alpha-2j} (1-C^2)^{j+(\alpha-|\alpha|)/2} \tag{52}$$

$$K(\gamma)_{\ell pq} = (-1)^{|q|}\, 2^\ell (1+\gamma)^{-\ell-|q|} \sum_{k=0}^{\infty} \sum_{r=0}^{|q|+k} \sum_{t=0}^{k} \frac{(-1)^r}{r!\,t!} \binom{2p-2\ell}{|q|+k-r}\binom{-2p}{k-t}\left(\frac{\ell-2p+q}{2}\right)^{r+t} \tag{53}$$

$$(1+\gamma)^{r+t-k} (1-\gamma)^k, \text{ (for } q > 0)$$

and

$$K(\gamma)_{\ell pq} = (-1)^{|q|}\, 2^\ell (1+\gamma)^{-\ell-|q|} \sum_{k=0}^{\infty} \sum_{r=0}^{|q|+k} \sum_{t=0}^{k} \frac{(-1)^t}{r!t!} \binom{-2p}{|q|+k-r}\binom{2p-2\ell}{k-t}\left(\frac{\ell-2p+q}{2}\right)^{r+t} \cdot \tag{54}$$

$$(1+\gamma)^{r+t-k} (1-\gamma)^k, \text{ (for } q < 0)$$

where

$$C = \cos\left(\frac{i}{2}\right)$$

$$k = \text{integral part of} \left[\frac{\ell - m}{2}\right]$$

$$j_1 = \max(0, -\alpha)$$

$$j_2 = \min(2\ell - 2p, \ell - m)$$

$$\alpha = m - \ell + 2p$$

The $\mathcal{R}_{\ell mpq}$ and $\mathrm{II}_{\ell mpq}$ functions in Equation 45 are given by

$$\mathcal{R}_{\ell mpq} = \sum_{n=0}^{k} \sum_{u=u_1}^{u_2} (-1)^{n+u} \delta_u \binom{|q|}{u} \binom{|\alpha|}{2n-u} \xi^{|q|-u} \eta^u P^{|\alpha|-2n+u} Q^{2n-u} \tag{55}$$

$$\mathrm{II}_{\ell mpq} = \sum_{n=0}^{k'} \sum_{u=u_1'}^{u_2'} (-1)^{n+u+1} \delta_u \binom{|q|}{u} \binom{|\alpha|}{2n+1-u} \xi^{|q|-u} \eta^u P^{|\alpha|-2n-1+u} Q^{2n+1-u} \tag{56}$$

where

$$k = \left[\frac{|q| + |\alpha|}{2}\right], \quad k' = \left[\frac{|q| + |\alpha| - 1}{2}\right]$$

$$u_1 = \max(0, 2n - |\alpha|), \quad u_2 = \min(2n, |q|)$$

$$u_1' = \max(0, 2n+1 - |\alpha|), \quad u_2' = \min(2n+1, |q|)$$

$\delta_u = 1$, if q, α are both positive or negative

$\delta_u = (-1)^u$, if q or α is negative

The integrals appearing in Equations 38 through 42 are given by the following expressions:

$$\int\left(\frac{\partial R_{20pq}}{\partial \lambda}\right) d\lambda = - J_2 \left(\frac{\mu a_e^2}{a^3}\right) J_{20p} K_{2pq} \left\{ \mathcal{R}_{20pq} \cos\theta_{20pq} + II_{20pq} \sin\theta_{20pq} \right\} \quad (57)$$

$$\int\left(\frac{\partial R_{20pq}}{\partial a}\right) d\lambda = \left(\frac{3}{2-2p+q}\right) J_2 \left(\frac{\mu a_e^2}{a^4}\right) J_{20p} K_{2pq} \left\{ \mathcal{R}_{20pq} \sin\theta_{20pq} - II_{20pq} \cos\theta_{20pq} \right\} \quad (58)$$

$$\int\left(\frac{\partial R_{20pq}}{\partial \xi}\right) d\lambda = - J_2 \left(\frac{\mu a_e^2}{a^3}\right) J_{20p} \left(\frac{1}{2-2p+q}\right) \left[\left\{ \left(\frac{\partial K_{2pq}}{\partial \xi}\right) \mathcal{R}_{20pq} + |q| K_{2pq} \mathcal{R}_{20pq}' \right\} \cdot \right. \quad (59)$$

$$\left. \sin\theta_{20pq} - \left\{ \left(\frac{\partial K_{2pq}}{\partial \xi}\right) II_{20pq} + |q| K_{2pq} II_{20pq}' \right\} \cos\theta_{20pq} \right]$$

$$\int\left(\frac{\partial R_{20pq}}{\partial \eta}\right) d\lambda = - J_2 \left(\frac{\mu a_e^2}{a^3}\right) J_{20p} \left(\frac{1}{2-2p+q}\right) \left[\left\{ \left(\frac{\partial K_{2pq}}{\partial \eta}\right) \mathcal{R}_{20pq} - qK_{2pq} II_{20pq}' \right\} \cdot \right. \quad (60)$$

$$\left. \sin\theta_{20pq} - \left\{ \left(\frac{\partial K_{2pq}}{\partial \eta}\right) II_{20pq} + q K_{2pq} \mathcal{R}_{20pq}' \right\} \cos\theta_{20pq} \right]$$

$$\int\left(\frac{\partial R_{20pq}}{\partial P}\right) d\lambda = - J_2 \left(\frac{\mu a_e^2}{a^3}\right) K_{2pq} \left(\frac{1}{2-2p+q}\right) \left[\left\{ \left(\frac{\partial J_{20p}}{\partial P}\right) \mathcal{R}_{20pq} \right. \right. \quad (61)$$

$$\left. + |2p-2| J_{20p} \mathcal{R}_{2m'pq} \right\} \sin\theta_{20pq} - \left\{ \left(\frac{\partial J_{20p}}{\partial P}\right) II_{20pq} \right.$$

$$\left. \left. + |2p-2| J_{20p} II_{2m'pq} \right\} \cos\theta_{20pq} \right]$$

and

$$\int\left(\frac{\partial R_{20pq}}{\partial Q}\right) d\lambda = - J_2 \left(\frac{\mu a_e^2}{a^3}\right) K_{2pq}\left(\frac{1}{2-2p+q}\right)\left[\left\{\left(\frac{\partial J_{20pq}}{\partial Q}\right) \mathcal{R}_{20pq} - (2p-2)\, J_{20p}\, \Pi_{2m'pq}\right\}\cdot\right. \tag{62}$$

$$\left.\sin\,\theta_{20pq} - \left\{\left(\frac{\partial J_{2op}}{\partial Q}\right) \Pi_{20pq} + (2p-2)\, J_{2op}\, \mathcal{R}_{2m'pq}\right\} \cos\,\theta_{20pq}\right]$$

where use has been made of the relations

$$\left.\begin{aligned}
&\frac{\partial \mathcal{R}_{\ell mpq}}{\partial \xi} = |q|\mathcal{R}_{\ell mpq'}\ , && \frac{\partial \Pi_{\ell mpq}}{\partial \xi} = |q|\ \Pi_{\ell mpq'} \\
&\frac{\partial \mathcal{R}_{\ell mpq}}{\partial \eta} = -\ q\ \Pi_{\ell mpq'}\ , && \frac{\partial \Pi_{\ell mpq}}{\partial \eta} = q\ \mathcal{R}_{\ell mpq'} \\
&\frac{\partial \mathcal{R}_{\ell mpq}}{\partial P} = |\alpha|\ \mathcal{R}_{\ell m'pq}\ , && \frac{\partial \Pi_{\ell mpq}}{\partial P} = |\alpha|\ \Pi_{\ell m'pq} \\
&\frac{\partial \mathcal{R}_{\ell mpq}}{\partial Q} = -\ \alpha\ \Pi_{\ell m'pq}\ , && \frac{\partial \mathcal{R}_{\ell mpq}}{\partial Q} = \alpha\mathcal{R}_{\ell m'pq}
\end{aligned}\right\} \tag{63}$$

and

$$q' = \begin{cases} q - 1 & (q > 0) \\ q + 1 & (q < 0) \end{cases} \tag{64}$$

$$m' = \begin{cases} -1 & (2p - 2 > 0) \\ +1 & (2p - 2 < 0) \end{cases} \tag{65}$$

The partial derivatives of the inclination and eccentricity functions are given by

$$\frac{\partial J_{\ell m p}}{\partial P} = -2P \; \frac{\partial J_{\ell m p}}{\partial C} \,, \tag{66}$$

$$\frac{\partial J_{\ell m p}}{\partial Q} = - \; 2Q \, \frac{\partial J_{\ell m p}}{\partial C} \,, \tag{67}$$

where

$$\frac{\partial J_{\ell m p}}{\partial C} = (-1)^k \; \frac{(\ell+m)!}{2^\ell p!(\ell-p)!} \sum_{j=j_1}^{j_2} \; (-1)^j \binom{2\ell - 2p}{j} \binom{2p}{\ell - m - j} \Big\{ C^{2\ell-\alpha-2j-1} \; .$$

$$S^{\alpha-|\alpha|} \; \Big[(2\ell-|\alpha|)S^{2j} - (2j+\alpha-|\alpha|)\,S^{2j-2}\Big] \Big\} \tag{68}$$

$$S = \sin\left(\frac{i}{2}\right) \tag{69}$$

and

$$\frac{\partial K_{\ell p q}}{\partial \xi} = - \; \frac{\xi}{\gamma} \; \frac{\partial K_{\ell p q}}{\partial \gamma} \tag{70}$$

$$\frac{\partial K_{\ell p q}}{\partial \eta} = - \; \frac{\eta}{\gamma} \, \frac{\partial K_{\ell p q}}{\partial \gamma} \tag{71}$$

where

$$\frac{\partial K_{\ell p q}}{\partial \gamma} = \frac{(-\ell-|q|)}{1+\gamma} \; K_{\ell p q} + (-1)^{|q|} 2^\ell (1+\gamma)^{-\ell-|q|} \sum_{k=0}^{\infty} \sum_{r=0}^{|q|+k} \sum_{t=0}^{k} \frac{(-1)^r}{r!t!} \; . \tag{72}$$

$$\binom{2p-2\ell}{|q|+k-r} \binom{-2p}{k-t} \left(\frac{\ell-2p+q}{2}\right)^{r+t} (1+\gamma)^{r+t-k-1} \; [(r+t-k)(1-\gamma)^k -$$

$$k(1+\gamma)(1-\gamma)^{k-1}], \quad (\text{for } q > 0)$$

$$\frac{\partial K_{\ell p q}}{\partial \gamma} = \frac{(-\ell-|q|)}{1+\gamma} K_{\ell p q} + (-1)^{|q|} 2^{\ell} (1+\gamma)^{-\ell-|q|} \sum_{k=0}^{\infty} \sum_{r=0}^{|q|+k} \sum_{t=0}^{k} \frac{(-1)^t}{r!t!} \cdot$$

$$\binom{-2p}{|q|+k-r} \binom{2p-2\ell}{k-t} \left(\frac{\ell-2p+q}{2}\right)^{r+t} (1+\gamma)^{r+t-k-1} [(r+t-k)(1-\gamma)^k - \tag{73}$$

$$k(1+\gamma)(1-\gamma)^{k-1}], \text{ (for } q < 0)$$

and

$$\frac{\partial K_{\ell p(2p-\ell)}}{\partial \gamma} = \frac{-2\ell+1}{\gamma} K_{\ell p(2p-\ell)} - 2\gamma^{-2\ell+2} \sum_{k=0}^{p'-1} \binom{\ell-1}{2k+|2p-\ell|} \cdot \tag{74}$$

$$\binom{2k+|2p-\ell|}{k} 2^{-2k-|2p-\ell|} k(1-\gamma^2)^{k-1}, \text{ (for } q = 2p-\ell \text{ and } p' = \frac{\ell-|2p-\ell|}{2})$$

It should be mentioned that even if convergence is not achieved (Equation 34), KMEC assumes that the final mean element values are correct and continues processing with them. This is done since near convegence may have occurred and the resulting mean elements may still be usable. Messages concerning the state of nonconvergence are generated to alert the user.

THE KEPLERIAN MEAN ELEMENT BUILDER (KEB)

FUNCTIONAL DESCRIPTION

The KEB function decomposes the mean nonsingular element set obtained from the WMI function into a mean Keplerian element set. The KEB process flow is shown in Figure 7.

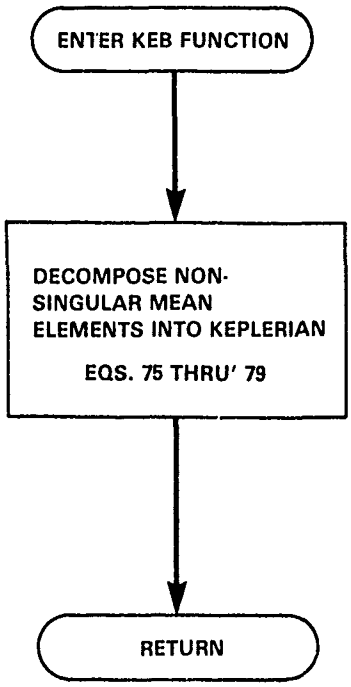

FIGURE 7. THE KEB FUNCTION PROCESS FLOW

PROCESSING EQUATIONS

The decomposition of the mean nonsingular element set into the mean Keplerian element set is accomplished through application of the following relations:

$$\bar{e} = (\bar{\xi}^2 + \bar{\eta}^2)^{1/2} \tag{75}$$

$$\bar{h} = \tan^{-1} (\bar{Q}/\bar{P}) \tag{76}$$

$$\bar{g} = \tan^{-1} (\bar{\eta}/\bar{\xi}) - \bar{h} \tag{77}$$

$$\bar{\ell} = \bar{\lambda} - (\bar{g} + \bar{h}) \tag{78}$$

and

$$\bar{i} = 2 \sin^{-1} [(\bar{P}^2 + \bar{Q}^2)^{1/2}] \tag{79}$$

Of course, no decomposition of the mean semimajor axis $\bar{a}$ is needed.

THE GEOCEIVER FORMAT SECTION (GFS)

FUNCTIONAL DESCRIPTION

The GFS function assembles the satellite ID, type, and mean Keplerian orbital elements; computes an earth-fixed longitude at epoch for the mean right ascension of the ascending node; and converts the epoch from modified Julian days to year, day, and minutes of day (GMT). These data are written to hard copy. The GFS process flow is illustrated in Figure 8.

PROCESSING EQUATIONS

The time of day in minutes (GMT) is computed by using the following:

$$t_{MIN} = \frac{(t_o - 367y + (7y/4) - d + 678957.)\,86400.}{60.} \tag{80}$$

where t_o is the epoch in modified Julian days, y is the year expressed as an integer, and d is the day of year. The right ascension of the Greenwich meridian L at epoch t_o is computed from

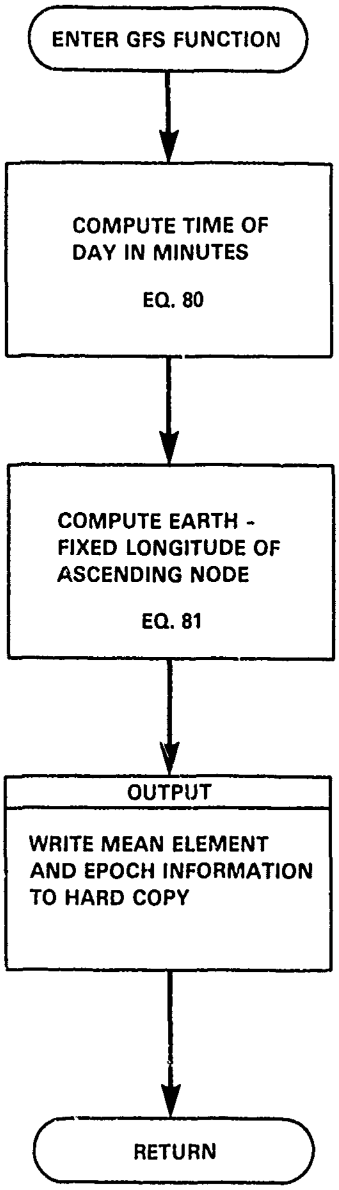

FIGURE 8. THE GFS FUNCTION PROCESS FLOW

$$L = \omega_{\oplus} (t_o - t_{ve}) \tag{81}$$

where $\omega_{\oplus}$ is the rotation rate of the earth and t_{ve} is the time of transit of the vernal equinox expressed in modified Julian days. The earth-fixed longitude Λ of the ascending node of the orbit is then computed by using

$$\Lambda = h - L \tag{82}$$

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APPENDIX

COMPUTER LISTING OF PROGRAM KMEC

09.44.26 03/04/83

```
      PROGRAM KMEC ( INPUT,OUTPUT,TAPE6=OUTPUT)
C
C
C
C     THIS PROGRAM IS THE KOZAI MEAN ELEMENT CONVERTER (KMEC) AND OPTIONALLY
C     CONVERTS BROUWER MEAN ELEMENTS OR OSCULATING CARTESIAN VECTORS TC
C     KOZAI MEAN ELEMENTS USING A NONSINGULAR ELEMENT FORMULATION.  A
C     DETAILED DESCRIPTION OF KMEC CAN BE FOUND IN THE DOCUMENT ENTITLED
C     " PROGRAM KMEC = THE COMPUTATION OF KOZAI MEAN ORBITAL ELEMENTS
C     USING A NONSINGULAR FORMULATION " BY A. D. PARKS.  ALL THE
C     EQUATION NUMBERS MENTIONED IN THIS PROGRAM REFER TO THOSE IN THIS
C     DOCUMENT .
C
C
C     INPUT GUIDE .......
C        CARD 1 ---- PROCESSING OPTION
C             IOAOP = 0       CONVERT BROUWER MEAN ELEMENTS
C             IOAOP = 1       CONVERT CARTESIAN VECTORS
C        CARD 2---- IDENTIFIER DATA
C             KSA = SATELLITE NUMBER
C             ITYPE = SATELLITE TYPE
C             FT = FREQUENCY OFFSET (FPM)
C             TVE = VERNAL EQUINOX TRANSIT TIME (MJD)
C     IF IOAOP = 0 , THEN CARD 3 THROUGH CARD 7 ARE THE FIVE CARD SPASUR
C     DATA.
C     IF ICAOP = 1, THEN----
C        CARD 3----VECTOR EPOCH
C             IAYR = YEAR
C             ADJDA = DAY OF YEAR
C             ADJSE = SECONDS OF DAY
C        CARD 4----POSITION VECTOR COMPONENTS
C             RV(1) = X
C             RV(2) = Y
C             RV(3) = Z
C        CARD 5----VELOCITY VECTOR COMPONENTS
C             RV(4) = XDOT
C             RV(5) = YDOT
C             RV(6) = ZDOT
C
C
C
C
      COMMON / CON / DEGRAD, XMU, XJ2, AE
      COMMON / KORBEL / BA, ES, XI, W, O, AM
      COMMON / MKEL / XM
      COMMCN / NORBEL / A, XL, Z, XN, P, Q
      DIMENSION XM(6)
      DATA XMU / 398600.8 /, XJ2/ 1082.6E-06 /, AE / 6378.135 /
      DATA R2 / 541.15E-06 /
      DATA B0,B2,B3,B4,B5 / 398600.8,-.1755528999E+11,.2638664773BE+12,
     1   .1063073996E+15,.8056050 22E+18 /
      DATA DT / 0.0 / , CN2 / 0.0 /, A1 / 0.0 /, E1 / 0.0 / , RN1/0.0/
C
C     THIS IS THE PFS FUNCTION
C
```

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```
      PI = 3.14159265359
      DEGRAD = PI / 180.
      READ *, IOAOP
      READ *, KSA, ITYPE, FT, TWE
C
C     ENTER CIS FUNCTION
C
      IF( IOAOP.NE. 0 ) CALL ORBADJ( IOAOP,KSA,UJD, IYLD,IDAY,SEC)
      IF ( IOAOP.NE.0) GO TO 40
      READ 4, KSAP, IYLD,IDAY
    4 FORMAT(2X,I5,40X,I1,I3)
      READ 5 , UJD, ETU, H0, G0, BES, BI
    5 FORMAT (8X,F14.8,5(1X,F8.4))
      READ 10, A0
   10 FORMAT (//8X,F11.5)
      BXI = BI*DEGRAD
      BW = G0*DEGRAD
      BO = H0*DEGRAD
      BAM = ETU*DEGRAD
C
C     SEE EQS. (2)-(3)
C
      FN = 1. / ( 1. - ES*ES ) ** 1.5
      TA = SIN( BXI )
      TB = TA * TA
      TE = 1. - ( 3.*TB)/2.
      TAD = (( 3. * R2)/(2.*A0*A0))*TE*FN
      BA = ( A0*((1. + 2.*TAD) / ( 1. - TAD )) ** 0.6666666667) * AE
      SEC = 0.00
      PRINT 15
   15 FORMAT (1H1 )
C
C     ENTER BIS FUNCTION
C
   30 CALL BRAUER (B0,B2,B3,B4,B5,DT,BA,BES,BXI,BAM,BW,BO,CN2,A,ES,XI,
     1     AM,W,O,A1,E1,RN1)
      PRINT 34
   34 FORMAT(/,56X,*BROUWER*)
      PRINT 35, KSA,UJD,A0,BA,BES,BI,BXI,ETU,BAM,G0,BW,H0,BO
   35 FORMAT    (39X,*MEAN ORBITAL ELEMENTS FOR SATELLITE *,I5,*:*
     1//9X,*EPOCH (JULIAN DAY MINUS 2,400,000.5)  *,E22.14/9X,
     2*SEMIMAJOR AXIS*,24X,E22.14,* EARTH RADII  *,E22.14,
     3* KILOMETERS*/9X,*ECCENTRICITY*,26X,E22.14/9X,
     4*INCLINATION*,27X,E22.14,* DEGREES*,6X,E22.14,* RADIANS*/9X,
     5*MEAN ANOMALY*,26X,E22.14,* DEGREES*,6X,E22.14,* RADIANS*/9X,
     6*ARGUMENT OF PERIGEE*,19X,E22.14,* DEGREES*,6X,E22.14,
     7* RADIANS*/9X,*RIGHT ASCENSION OF THE ASCENDING NODE *,E22.14,
     8* DEGREES*,6X,E22.14,* RADIANS*/)
      BA = A
C
C     ENTER NEB FUNCTION
C
   40 CALL FORM
C
C     ENTER WHI FUNCTION
```

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```
C
      CALL MEAN
      XXXL = XM(2) / DEGRAD
      PRINT 15
      PRINT 50, KSA,UJD,XM(1),XXXL,XM(3),XM(4),XM(5),XM(6)
   50 FORMAT(28X,*MEAN NONSINGULAR ORBITAL ELEMENTS FOR SATELLITE *,I5,
     1* - *,//31X,*EPOCH (JULIAN DAY MINUS 2,400,000.5 ) *,E22.14/31X,
     2*SEMIMAJOR AXIS*,24X,E22.14,* KILOMETERS*/31X,*LAMBDA*,32X,E22.14,
     21X,
     3*DEGREES*/31X,*ZETA*,34X,E22.14/31X,*ETA*,35X,E22.14/31X,*P*,37X,
     4 E22.14/31X,*Q*,37X,E22.14/)
C
C     ENTER KEB FUNCTION
C
      CALL DCMPOS
      XA = BA / AE
      XXI = XI / DEGRAD
      XW = W / DEGRAD
      XO = O / DEGRAD
      XAM = AM / DEGRAD
      PRINT 60
   60 FORMAT (///,57X,*KOZAI*)
      PRINT 35, KSA,UJD,XA,BA,ES,XXI,XI,XAM,AM,XW,W,XO,O
C
C     ENTER GFS FUNCTION
C
      CALL MX1502(KSA,ITYPE,FT,IYLD,IDAY,BA,XXI,ES,XW,XO,XAM,UJD,TWE,
     1              SEC,IOAOP)
      STOP
      END
      SUBROUTINE FORM
C
C
C     THIS IS THE NEB FUNCTION.
C     OSCULATING NONSINGULAR ELEMENTS ARE FORMED FROM THE OSCULATING
C     KEPLERIAN ELEMENTS.  SEE EQS.  (1).
C
C
      COMMON / KORBEL / BA, ES, XI, W, O, AM
      COMMON / NORBEL / A, XL, Z, XN, P, Q
      PI = 3.14159265359
      PI2 = 2.*PI
      A = BA
      XL = W + O + AM
      XL = AMOD( XL,PI2 )
      WB = W + O
      WB = AMOD( WB,PI2 )
      Z = ES * COS( WB )
      XN = ES * SIN( WB )
      P = SIN(0.5*XI) * COS( O )
      Q = SIN(0.5*XI)*SIN( O )
      PRINT 9
    9 FORMAT(/,46X,*OSCULATING NONSINGULAR ELEMENT SET--*)
      PRINT 10, A, XL, Z, XN, P, Q
   10 FORMAT(/,50X,*A     =*,G16.10,* KM*,/50X,
```

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```
     1              *LAMBDA=*,G16.10,*RAD*,/50X,
     2              *ZETA   =*,G16.10,/50X,
     3              *ETA    =*,G16.10,/50X,
     4              *P      =*,G16.10,/50X,
     5              *Q      =*,G16.10)
      RETURN
      END
      SUBROUTINE MEAN
C
C
C     THIS IS THE WMI FUNCTION.  MEAN NONSINGULAR ELEMENTS ARE OBTAINED
C     USING THE WALTER ALGORITHM.
C
C
      DIMENSION XOSC(6),XM(6),TOL(6),DELX(6),XSP(6),XMN(6)
      DIMENSION SUM(6)
      COMMON / NORBEL / A,XL,Z,XN,P,Q
      COMMON / MNEL / XM
      COMMON / INTG / SUM, G, XMOT
      DATA TOL / 0.01,5*0.00001 /
      XOSC(1) = A
      XOSC(2) = XL
      XOSC(3) = Z
      XOSC(4) = XN
      XOSC(5) = P
      XOSC(6) = Q
      KOUNT = 0
C
C     INITIALIZE PROCESS.  SEE EQ. (35)   .
C
      DO 10  I = 1,6
   10 XM(I) = XOSC(I)
C
C     EVALUATE SUMS OF INTEGRALS OF GEOPOTENTIAL DISTURBING FUNCTION
C     PARTIAL DERIVATIVES.  SEE EQS. (57)-(74),(37)-(42).
C
   20 CALL EVI
C
C     LOAD SHORT PERIODIC ARRAY.
C
      XSP( 1 ) = XSPA ( XM )
      XSP(2) = XSPL( XM, SUM, G, XMOT )
      XSP(3) = XSPZ( XM, SUM, G, XMOT )
      XSP(4) = XSPXN( XM, SUM, G, XMOT )
      XSP(5) = XSPP( XM, SUM, G, XMOT )
      XSP(6) = XSPQ( XM, SUM, G, XMOT )
      KOUNT = KOUNT + 1
C
C     ITERATE FOR MEAN ELEMENTS.  SEE  EQS. (33)-(34).
C
      DO 50  J = 1, 6
      XMN(J) = XOSC(J) - XSP(J)
      DELX(J) = ABS( XMN(J) - XM(J))
      IF ( DELX(J) .LE. TOL(J)) GO TO 50
   40 XM(J) = XMN(J)
```

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```
   50 CONTINUE
      DO 60  K = 1,6
      IF(DELX(K).GT.TOL(K).AND.KOUNT.LT.100) GO TO 20
   60 CONTINUE
   70 IF(KOUNT .LT. 100 ) GO TO 90
      WRITE (6,100)  KOUNT
  100 FORMAT(/,24X,*THE KOZAI MEAN ELEMENT CONVERSION ALGORITHM DID NOT
     1 CONVERGE IN *,I5,* ITERATIONS.*)
      PRINT 101,  (TOL(I),I=1,6),(DELX(I),I=1,6)
  101 FORMAT(/,5X,*THE DESIRED TOLERANCES WERE--*,/,6(2X,E12.4)/5X,
     2   *THE FINAL TOLERANCES  WERE--*,/,6(2X,E12.4))
      RETURN
   90 CONTINUE
      PRINT 91, KOUNT
   91 FORMAT(/,30X,*THE KOZAI MEAN ELEMENT CONVERSION ALGORITHM CONVERGE
     1D IN *,I5,* ITERATIONS.*)
      RETURN
      END
      SUBROUTINE EVI
C
C
C     THIS IS PART OF THE WMI FUNCTION.  SUMMATIONS OVER P AND Q OF
C     THE INTEGRALS OF THE GEOPOTENTIAL DISTURBING FUNCTION PARTIALS ARE
C     EVALUATED.  SEE EQS. (6.26) - (57)-(74),(37)-(42).
C
C
      DIMENSION XM(6), SUM(6)
      COMMON / INTG / SUM, G, XMCT
      COMMON / CON / DEGRAD, XMU, XJ2, AE
      COMMON / MNEL / XM
      COMMON / KORBEL / AA,ES,XI,W,OM,AM
      XMOT = SQRT( XMU/ ( XM(1)**3))
      L = 2
      M = 0
      DO 10   I = 1, 6
      SUM(I) = 0.0
   10 CONTINUE
      CALL DCMPOS
      AA = XM(1)
      G = 1. - ES*ES
      G = SQRT(G)
      A = XM(3)
      B = XM(4)
      C = XM(5)
      D = XM(6)
      CM = -XJ2*(XMU*AE*AE/(AA**3))
      CM1 = -CM/AA
      DO 20    I = 1, 3
      IP = I - 1
      IQT = 2*IP - 2
      IAL = M - L + 2*IP
      DO 30    J = 1, 5
      IQ = J - 3
      IF(IQ.EQ.IQT) GO TO 30
      THT = (L-2*IP+IQ)*XM(2)
```

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```
      IQP = 0
      IF(IQ.GT.0) IQP = IQ - 1
      IF ( IQ.LT. 0 ) IQP = IQ + 1
      MP = 0
      IF ( IAL .LT. 0 ) MP = M + 1
      IF ( IAL . GT . 0 ) MP = M - 1
      CJ = XJLMP(L,M,IP,XI)
      CK = XKLPQ(L,IP,IQ,G)
      DKDZ = - ( XM(3)/G)*DKLPQ(L, IP, IQ, G)
      DKDN = - ( XM(4)/G)*DKLPQ(L, IP, IQ, G)
      DJDP = -2.*XM(5)*DJLMP(L,M,IP,XI)
      DJDQ = -2.*XM(6)*DJLMP(L,M,IP,XI)
      R = RLMPQ(L,M,IP,IQ,A,B,C,D)
      RQP = RLMPQ(L,M,IP,IQP,A,B,C,D)
      RMP = RLMPQ(L,MP,IP,IQ,A,B,C,D)
      BI = BILMPQ(L,M,IP,IQ,A,B,C,D)
      BIQP = BILMPQ(L,M,IP,IQP,A,B,C,D)
      BIMP = BILMPQ(L,MP,IP,IQ,A,B,C,D)
      CT = COS(THT)
      ST = SIN(THT)
      SUM(1) = SUM(1) + CM*CJ*CK*(R*CT+BI*ST)
      SUM(2) = SUM(2) + (3./(2.-2.*IP+IQ))*CM1*CJ*CK*(R*ST-BI*CT)
      SUM(3) = SUM(3) + (1./(2.-2.*IP+IQ))*CM*CJ*((DKDZ*R+IABS(IQ)*CK
     1   *RQP)*ST-(DKDZ*BI+IABS(IQ)*CK*BIQP)*CT)
      SUM(4) = SUM(4) + (1./(2.-2.*IP+IQ))*CM*CJ*((DKDN*R-IQ*CK*BIQP)*ST
     1   - (DKDN*BI +IQ*CK*RQP)*CT)
      SUM(5) = SUM(5)+(1./(2.-2.*IP+IQ))*CM*CK*((DJDP*R+IABS(2*IP-2)*
     1   CJ*RMP)*ST - (DJDP*BI+IABS(2*IP-2)*CJ*BIMP)*CT)
      SUM(6) = SUM(6) + (1./(2.-2.*IP+IQ))*CM*CK*((DJDQ*R-(2.*IP-2)*
     1   CJ*BIMP)*ST-(DJDQ*BI+(2*IP-2)*CJ*RMP)*CT)
   30 CONTINUE
   20 CONTINUE
      RETURN
      END
      SUBROUTINE DCMPOS
C
C
C     THIS IS THE KEB FUNCTION.  MEAN NONSINGULAR ELEMENTS ARE
C     DECOMPOSED INTO MEAN KEPLERIAN ELEMENTS.  SEE EQS. (75)-(79).
C
C
      DIMENSION XM(6)
      COMMON / MNEL / XM
      COMMON / KORBEL / A,ES,XI,W,OM,AM
      PI = 3.14159265359
      PI2 = 2.*PI
      ES = SQRT(XM(3)*XM(3) + XM(4)*XM(4))
      OM = ARTNQ(XM(6),XM(5))
      WB = ARTNQ(XM(4),XM(3))
      W = WB - OM
      IF ( W.LT. 0.0) W = PI2 + W
      XI = 2.*ASIN(SQRT(XM(5)*XM(5)+XM(6)*XM(6)))
      A = XM(1)
      AM = XM(2) - WB
      IF   AM.LT. 0.0) AM = PI2 + AM
```

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```
      RETURN
      END
      SUBROUTINE BRAUER(B0,B2,B3,B4,B5,DT,A2P,E2P,CI2P,CL02P,G02P,H02P,
     1CN2,A,CE,CI,CL,G,H,ADT,RND,ESD)
C
C
C     THIS IS THE BIS FUNCTION.  A DRAG AUGMENTED BROUWER - LYDDANE
C     THEORY IS USED TO GENERATE OSCULATING KEPLERIAN ELEMENTS FROM
C     MEAN BROUWER ELEMENTS.  SEE EQS. (4)-(25).
C
C
 1004 A2P2=A2P**2
      A2P4=A2P2**2
      CN0=SQRT(B0/(A2P2*A2P))
      E2P2=E2P**2
      ETA=SQRT(1.-E2P2)
      SINEI=SIN(CI2P)
      THETA=COS(CI2P)
      THETA2=THETA**2
      THETA4=THETA2**2
      THETA6=THETA4*THETA2
      CJ2=-B2/(2.*B0*A2P2)
      ETA2=ETA**2
      ETA3=ETA2*ETA
      ETA4=ETA2**2
      CJ21P=CJ2/ETA4
      CJ31P=B3/(B0*A2P2*A2P*ETA4*ETA2)
      CJ41P=(3.*B4)/(8.*B0*A2P4*ETA4*ETA4)
      CJ51P=B5/(B0*A2P4*A2P*ETA4**2*ETA2)
      FUN1=3.*THETA2-1.
      FUN2=1.-5.*THETA2
      SINEI2=SINEI**2
      A1=A2P*CJ2*FUN1
      A0=-A1/ETA3
      A2=3.*A2P*CJ2*SINEI2
      FUN5=1.-11.*THETA2-(40.*THETA4)/FUN2
      FUN6=        -FUN1-(8.*THETA4)/FUN2
      FUN4=THETA2/SINEI2
      FUN22=FUN2**2
      CJ21P2=CJ21P**2
      E01P=((E2P*ETA2)*(3.*CJ21P2*FUN5-10.*
     1CJ41P*FUN6))/(24.*CJ21P)
      E21P=-2.*E01P
      E31P=((35.*CJ51P*E2P2*ETA2*SINEI)*(FUN2-(16.*THETA4)/FUN2))/(96.*
     1CJ21P)
      E11P=-.75*E31P+((.25*ETA2*SINEI)*(CJ31P+.3125*CJ51P*(4.+3.*E2P2)*
     1(1.-9.*THETA2-(24.*THETA4)/FUN2)))/CJ21P
      CI0=-(E2P*THETA)/(ETA2*SINEI)
      CI2=CJ21P*THETA*SINEI*1.5
      CI1=E2P*CI2*.66666667
      FUN7=(-.5*ETA3*CJ21P)/E2P
      CL21P=(ETA3/CJ21P)*(.25*CJ21P2*FUN5-.83333333*CJ41P*FUN6)
      CL12P=CN0*(1.+1.5*CJ21P*ETA*FUN1+.09375*CJ21P2*ETA*(-15.+16.*ETA+
     125.*ETA2+(30.-96.*ETA-90.*ETA2)*THETA2+(105.+144.*ETA+25.*ETA2)*
     2THETA4)+.9375*CJ41P*ETA*E2P2*(3.-30.*THETA2+35.*THETA4))
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      CL22P=.5*CN0*CN2
      G21P=(1./(24.*CJ21P))*(-3.*CJ21P2*(2.+E2P2-11.*(2.+3.*E2P2)*THETA2
     1-40.*(2.+5.*E2P2)*THETA4/FUN2-400.*E2P2*THETA6/FUN22)+10.*CJ41P*
     2(2.
     3+E2P2-3.*(2.+3.*E2P2)*THETA2-8.*(2.+5.*E2P2)*THETA4/FUN2-80.*E2P2*
     4THETA6/FUN22))
      G12P=CN0*(-1.5*CJ21P*FUN2+.09375*CJ21P2*(-35.+24.*ETA+25.*ETA2+
     1(90.-192.*ETA-126.*ETA2)*THETA2+(385.+360.*ETA+45.*ETA2)*THETA4)+
     2.3125*CJ41P*(21.-9.*ETA2+(-270.+126.*ETA2)*THETA2+(385.-189.*ETA2)
     3*THETA4))
      H2=1.5*CJ21P*THETA
      H3=-2.*H2
      H1=.66666667*E2P*H2
      H31P=((35.*CJ51P*E2P2*E2P*THETA)/(144.*CJ21P))*(.5/SINEI*(FUN2-(
     116.*THETA4)/FUN2)+SINEI*(5.+(32.*THETA2)/FUN2+80.*THETA4/FUN22))
      H11P=-.25*
     1          H31P+((.25*E2P*THETA)/(CJ21P*SINEI))*(CJ31P+.3125*CJ51P*
     2(4.+3.*E2P2)*(1.-9.*THETA2-(24.*THETA4)/FUN2)+1.875*CJ51P*SINEI2*
     3(4.+3.*E2P2)*(3.+(16.*THETA2)/FUN2+(40.*THETA4)/FUN22))
      H21P=(E2P2*THETA)/(12.*CJ21P)*(-3.*CJ21P2*(11.+(80.*THETA2)/FUN2+
     1(200.*THETA4)/FUN22)+10.*CJ41P*(3.+(16.*THETA2)/FUN2+(40.*THETA4)/
     2FUN22))
      H12P=CN0*THETA*(-3.*CJ21P+.375*CJ21P2*(-5.+12.*ETA+9.*ETA2+(-35.-
     136.*ETA-5.*ETA2)*THETA2)+1.25*CJ41P*(5.-3.*ETA2)*(3.-7.*THETA2))
      AID=CJ51P/CJ21P
      AID2=FUN2-(16.*THETA4)/FUN2
      C1=35./384.*AID*ETA3*E2P*SINEI*AID2
      AID3=THETA2/SINEI
      AID4=THETA2*SINEI
      E2P3=E2P2*E2P
      C2=35./1152.*AID*((-E2P*SINEI*(3.+2.*E2P2)+E2P3*AID3)*AID2+
     112.*E2P3*AID4*(5.+(32.*THETA2)/FUN2+(80.*THETA4)/FUN22))
      C3=1.-9.*THETA2-(24.*THETA4)/FUN2
      AID5=CJ31P/CJ21P
      C4=.25*AID5*(-E2P*AID3)+5./64.*AID*(-E2P*AID3*(4.+3.*E2P2)+
     1E2P*SINEI*(26.+9.*E2P2))*C3-15./32.*AID*E2P*AID4*(4.+3.*E2P2)*
     2(3.+(16.*THETA2)/FUN2+(40.*THETA4)/FUN22)
      C5=E2P/(1.+ETA3)*(3.-E2P2*(3.-E2P2))
      C6=(E2P*(-32.+81.*(E2P2*E2P2)))/((4.+3.*E2P2)+ETA*(4.+9.*E2P2))
      C7=.25*AID5*SINEI*C5+5./64.*C3*AID*ETA2*SINEI*C6
      C8=-.25*AID5*ETA3*SINEI-5./64.*AID*ETA3*SINEI*(4.+9.*E2P2)*C3
 0510 T=DT
      CL2P=CL12P*DT+CL22P*DT**2+CL02P + RN0*DT*DT
      CL2P=AMOD(CL2P,6.2831853071796)
      IF(CL2P)520,530,530
  520 CL2P=CL2P+6.2831853071796
  530 G2P=G12P*DT+G02P
      H2P=H12P*DT+H02P
      SINEG=SIN(G2P)
      COSING=COS(G2P)
      D1E=SINEG*(SINEG*(E31P*SINEG+E21P)+E11P)+E01P
      H1P=((H31P*SINEG+H21P)*SINEG+H11P)*COSING+H2P
      GPLP=G2P+CL2P+.5*(CL21P+G21P)*SIN(2.*G2P)+(C1+C2)*COS(3.* G2P)
     1+(C4+C7)*COSING
      CL1P=CL2P
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      U=CL2P
  100 DELTAU=(U-E2P*SIN(U)-CL2P)/(1.-E2P*COS(U))
      U=U-DELTAU
      IF(ABS(DELTAU)-1.E-10)200,100,100
  200 U=U-(U-E2P*SIN(U)-CL2P)/(1.-E2P*COS(U))
      E=U
      SINE1P=SIN(E)
      COSE1P=COS(E)
      G1P=G2P
      ADIVR=1./(1.-E2P*COSE1P)
      SINF1P=ADIVR*ETA*SINE1P
      COSF1P=ADIVR*(COSE1P-E2P)
      F1P=ARTNQ(SINF1P,COSF1P)
      IF(ABS(F1P-CL2P)-3.14159265358 98)220,210,210
  210 STOP
  220 FUN3=(1.+CN2*T)**.66666667
      COSFG=COS(2.*(G1P+F1P))
      SINFG=SIN(2.*(G1P+F1P))
      ADIVR2=ADIVR**2
      ADIVR3=ADIVR**3
      CI=CI2P+CI0*D1E+CI1*SINF1P*SINFG+(2.*CI1*CCSF1P+CI2)*COSFG
      FUN8=F1P-CL1P+E2P*SINF1P
 8018 H=H1P+(2.*H1*COSF1P+H2)*SINFG-H1*SINF1P*COSFG+H3*=UN8
      KFUN=H/6.2831853071796
      FUN9=KFUN
      H=H-FUN9*6.2831853071796
      IF(H)8022,8023,8023
 8022 H=H+6.2831853071796
 8023 A=A2P/FUN3+A0+(A1+A2*COSFG)*ADIVR + ADT*DT
      AID6=ADIVR2*ETA2+ADIVR
      AID7=SIN(2.*G2P+F1P)
      AID8=SIN(2.*G2P+3.*F1P)
      D1=.25*CJ21P*(6.*(5.*THETA2-1.)*FUN8+(3.-5.*THETA2)*(3.*SINFG+
     13.*E2P*AID7 +E2P*AID8 ))
      D2=.25*CJ21P*(2.*(3.*THETA2-1.)*(AID6+1.)*SINF1P+3.*(1.-THETA2)*
     1    ((-AID6+1.)*AID7+(AID6+.33333333)*AID8))
      AID9=COS(2.*G2P+F1P)
      AID10=COS(2.*G2P+3.*F1P)
      D3=-ETA2*.5*CJ21P*(1.-THETA2)*(3.*AID9+AID10)
      ETA6I=1./(ETA3*ETA3)
      D4=ETAEI*(C5 +COSF1P*(3.+E2P*COSF1F*(3.+E2P*COSF1P)))
      D5=ETA6I*(E2P+COSF1P*(3.+E2P*COSF1P*(3.+E2P*COSF1P)))
      D6= ETA2*CJ2*.5*((3.*THETA2-1.)*D4+3.*(1.-THETA2)*D5*COSFG)+D3
      GAL=GPLP+D1+(E2P*ETA2)/(1.+ETA)*D2
      CE=(E2P-1.)*(1.+CN2*DT)**.66666666666667+1.+D1E+D6 + ESD*DT
      EDL=.5*E2P*CL21P*SIN (2.*G2P)+C8*COSING+E2P*C1*COS (3.*G2P)-
     1 ETA3*D2
      AID14=SIN(CL2P)
      AID15=COS(CL2P)
      ESL=CE *AID14+EDL*AID15
      ECL=CE *AID15-EDL*AID14
      CE=SQRT(ECL*ECL+ESL*ESL)
      CL=ARTNQ(ESL,ECL)
      G=GAL-CL
      G=AMOD(G,6.2831853071796)
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      IF(G)8024,8025,8025
 8024 G = G + 6.2831853071796
 8025 RETURN                                                            BRLY1560
      END                                                               BRLY1570
      SUBROUTINE CORDEL(RV,HV,Q,TAUE,U)
C
C
C     THIS IS PART OF THE CIS FUNCTION.  OSCULATING CARTESIAN POSITION
C     AND VELOCITY VECTORS ARE TRANSFORMED INTO OSCULATING KEPLERIAN
C     ELEMENTS.   SEE EQS.  (26)-(31).
C
C
      DIMENSION RV(6),HV(6)
      X=RV(1)
      Y=RV(2)
      Z=RV(3)
      XDOT=RV(4)
      YDOT=RV(5)
      ZDOT=RV(6)
       R=SQRT(X*X+Y*Y+Z*Z)
      VSQ=XDOT*XDOT+YDOT*YDOT+ZDOT*ZDOT                                EL  0060
      HX=Y*ZDOT-Z*YDOT
      HY=Z*XDOT-X*ZDOT
      HZ=X*YDOT-Y*XDOT
      H=SQRT(HX*HX+HY*HY+HZ*HZ)
      AA=1./(2./R-VSQ/Q)
      ESINU=(X*XDOT+Y*YDOT+Z*ZDOT)/SQRT(Q*AA)
      ECOSU=R*VSQ/Q-1.
      ESQ=ESINU*ESINU+ECOSU*ECOSU
      E=SQRT(ESQ)
      ROOT=SQRT(1.-ESQ)
      ANGLEI=ARTNQ(SQRT(HX*HX+HY*HY),HZ)
      IF(ANGLEI-TAUE)1,5,5
    1 SGNHZ=HZ/ABS(HZ)
      ANGLEI=1.5707963267949*(1.-SGNHZ)
      OMEGA=0.
      IF(E-TAUE)2,3,3
    2 ANOMAL=ARTNQ(SGNHZ*Y,X)
      GO TO 9
    3 PERIG=ARTNQ(SGNHZ*Y,X)-ARTNQ(ROOT*ESINU,ECOSU-ESQ)
      IF(PERIG)4,7,7
    4 PERIG=PERIG+6.2831853071796
      GO TO 7
    5 OMEGA = ARTNQ(HX,-HY)
      IF(E-TAUE)8,6,6
    6 PERIG=ARTNQ(Z*H*(ECOSU-ESQ)+(X*HY-Y*HX)*ROOT*ESINU,Z*H*ROOT*
     1ESINU-(X*HY-Y*HX)*(ECOSU-ESQ))
    7 U=ARTNQ(ESINU,ECOSU)
      ANOMAL=U-ESINU
      GO TO 10
    8 ANOMAL=ARTNQ(Z*H,Y*HX-X*HY)
    9 E = 0.
      PERIG=0.
   10 CONTINUE
C     TAU=-ANOMAL*SQRT(AA*AA*AA/Q)/3600.
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      TAU=ANOMAL
      HV(1)=AA
      HV(2)=E
      HV(3)=ANGLEI
      HV(4)=PERIG
      HV(5)=OMEGA
      HV(6)=TAU
      RETURN
      END
      SUBROUTINE ORBADJ( IOAOP, KSA, TOA , IAYR, IDJDA, ADJSE )
C
C
C     THIS IS PART OF THE CIS FUNCTION.
C
C
      COMMON / KORBEL / A01,E1,XI1,G01,H01,FL01
      DIMENSION RV(6) , HV(6)
      IF ( IOAOP . EQ . 0 ) RETURN
      READ * , IAYR, ADJDA, ADJSE
      READ *, RV(1), RV(2), RV(3)
      READ * , RV(4) , RV(5), RV(6)
      TOA = 1721044. + 367*IAYR - (7*IAYR)/4 + ADJDA + (ADJSE/86400.)
     1    - 2400001.0
      Q = 398600.8
      TAUE = 1.E-06
      CALL CORDEL ( RV,HV,Q,TAUE,U )
      A01 = HV(1)
      E1 = HV(2)
      XI1 = HV(3)
      G01 = HV(4)
      H01 = HV(5)
      FL01 = HV(6)
      PRINT 15
   15 FORMAT ( 1H1 )
   25 PRINT 20, IOAOP
   20   RMAT(42X,*PROCESSING OPTION *,I4,* SELECTED*,/)
      PRINT 10, IAYR, ADJDA, ADJSE
   10 FORMAT   (39X,*THE POST ORBIT ADJUST EPOCH IS-  YEAR*,I5,*  DAY *,
     1   F6.1,*  SEC  *,G16.10)
      IDJDA = ADJDA
      PRINT 11
   11 FORMAT (/,39X,*INPUT POST ORBIT ADJUST CARTESIAN VECTORS ARE --*)
      PRINT 12, ( RV(I), I=1,6 )
   12 FORMAT(/,39X,*X    =*,G16.10,* KM*/39X,
     1               *Y    =*,G16.10,* KM*/39X,
     2               *Z    =*,G16.10,* KM*/39X,
     3               *XDOT=*,G16.10,* KM PER SEC*/39X,
     4               *YDOT=*,G16.10,* KM PER SEC*/39X,
     5               *ZDOT=*,G16.10,* KM PER SEC*)
      RETURN
      END
      SUBROUTINE MX1502(KSA,ITYPE,FT,IY,ID,BA,XXI,ES,XH,XO,XAM,UJD,TVE,
     1                  SEC, IOIOP)
C
C
```

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```
C     THIS IS THE GFS FUNCTION.  THE FOLLOWING SHOULD BE NOTED=
C        (1) THE INPUT VALUE FOR TVE SHOULD BE PERIODICALLY UPDATED
C            FROM THE NAUTICAL ALMANAC.
C        (2) THE YEAR COMPUTATION IAYR ASSUMES THAT ALL VECTOR EPOCHS
C            ARE BETWEEN THE YEARS 1980 AND 1989 .  EPOCH YEARS OTHER
C            THAN THESE WILL NECESSITATE CHANGING 1980 IN THE IAYR
C            EXPRESSION TO THE APPROPRIATE DECADE.
C
C
      DATA WE / 7.292115856E-05 /
      PRINT 10
   10 FORMAT ( 1H1 )
      PI = 3.14159265359
      PI2 = 2.*PI
      DEG = PI / 180.
      IF ( IOAOP .EQ. 0 ) GO TO 15
      IAYR = IY
      TM = SEC / 60.
      GO TO 18
C
C     COMPUTE MINUTES OF DAY (GMT) .  SEE EQ. (80).
C
   15 IAYR = 1980 + IY
      TS = UJD - 1721044. - 367*IAYR + (7*IAYR)/4 - ID + 2400001.0
      TM = (TS * 86400.) / 60.
C
C     COMPUTE EARTH FIXED LONGITUDE OF THE ASCENDING NODE.  SEE EQ. (81)
C
   18 RAG = WE * ( UJD - TVE ) * 86400.
      RAG = AMOD ( RAG, PI2)
      RAG = RAG / DEG
      XOL = XO - RAG
      IF ( XOL.LT. 0.00 ) XOL = 360. + XOL
      PRINT 20
   20 FORMAT(////,37X,*GEODETIC SATELLITE ORBIT PARAMETERS FOR THE MX 15
     102-DS GEOCEIVER*)
      PRINT 30, KSA,ITYPE,IAYR,ID,TM,XAM,XW,ES,BA,XOL,XXI,FT
   30 FORMAT(//,57X,*SATELLITE IDENTIFICATION*,6X,I5/57X,
     1   *SATELLITE TYPE*,16X,I5/57X,
     2   *ELEMENT SET EPOCH (GMT)*,/76X,
     3        *YEAR*,7X,I5/77X,
     4        *DAY*,7X,I5/76X,
     5        * MIN*,5X,F7.2///44X,
     6   *MEAN ANOMALY*,20X,E22.14,* DEG*/44X,
     7   *ARGUMENT OF PERIGEE*,13X,E22.14,* DEG*/44X,
     8   *ECCENTRICITY*,20X,E22.14/44X,
     9   *SEMI-MAJOR AXIS*,17X,E22.14,* KM*/44X,
     A   *LONGITUDE OF ASCENDING NODE*,5X,E22.14,* DEG*/44X,
     B   *INCLINATION*,21X,E22.14,* DEG*/44X,
     C   *TRANSMISSION FREQUENCY*,10X,E22.14,* PPM*/)
      RETURN
      END
      FUNCTION XSPA(XM)
C
C
```

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```
C     COMPUTE THE SHORT PERIODIC VARIATION OF THE SEMI-MAJOR AXIS.
C     SEE EQ. (37).
C
C
      DIMENSION XM(6)
      COMMON / KORBEL / A, ES, XI, W, OM, AM
      COMMON / CON / DEGRAD, XMU, XJ2, AE
      F = AM + 2.*ES*SIN(AM)+(5./4.)*ES*ES*SIN(2.*AM)+(13./12.)*ES*ES*
     1   SIN(3.*AM)
      P = XM(1) * ( 1. - ES*ES )
      RR= P / ( 1. + ES*COS(F))
      XSPA = XJ2*(AE*AE/XM(1))*(((XM(1)/RR)**3.)*(1.-1.5*SIN(XI)*
     1   SIN(XI)+1.5*SIN(XI)*SIN(XI)*COS(2.*W+2.*F))-(1.-1.5*SIN(XI)*
     1   SIN(XI))*(1.-ES*ES)**(-1.5))
      RETURN
      END
      FUNCTION XSPL    ( XM, SUM, G, XMOT )
C
C
C     COMPUTE THE SHORT PERIODIC VARIATION OF LAMBDA.  SEE EQ. (38).
C
C
      DIMENSION SUM(6), XM(6)
      XSPL =        - (2./(XMOT*XMOT*XM(1)))*SUM(2)+(G/(2.*XMOT*XMOT*XM(1
     1   )*XM(1)))*(XM(3)*SUM(3)+XM(4)*SUM(4))+(1./(2.*XMOT*XMOT*XM(1)*
     2   XM(1)*G))*(XM(5)*SUM(5)+XM(6)*SUM(6))
      RETURN
      END
      FUNCTION XSPZ    ( XM, SUM, G, XMOT )
C
C
C     COMPUTE THE SHORT PERIODIC VARIATION OF XI.  SEE EQ. (39).
C
C
      DIMENSION SUN(6), XM(6)
      XSPZ = -(G/(XMOT*XMOT*XM(1)*XM(1)*(1.+G)))*XM(3)*SUM(1)-(G/(XMOT*
     1   XMOT*XM(1)*XM(1)))*SUM(4)-(1./(2.*XMOT*XMOT*XM(1)*XM(1)*G))*
     2   XM(4)*(XM(5)*SUM(5)+XM(6)*SUM(6))
      RETURN
      END
      FUNCTION XSPXN  ( XM, SUM, G, XMOT )
C
C
C     COMPUTE THE SHORT PERIODIC VARIATION OF ETA.  SEE EQ. (40).
C
C
      DIMENSION SUM(6), XM(6)
      XSPXN=-(G/(XMOT*XMOT*XM(1)*XM(1)*(1.+G)))*XM(4)*SUM(1)+(G/(XMOT*
     X   XMOT*
     1   XM(1)*XM(1)))*SUM(3) + (1./(2.*XMOT*XMOT*XM(1)*XM(1)*G))*XM(3)
     2   *(XM(5)*SUM(5)+XM(6)*SUM(6))
      RETURN
      END
      FUNCTION XSPP    ( XM, SUM, G, XMOT )
C
```

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```
C
C     COMPUTE THE SHORT PERIODIC VARIATICN OF F.  SEE EQ. (41).
C
C
      DIMENSION SUM(6), XM(6)
      C1 = 1./(2.*XMOT*XMOT*XM(1)*XM(1)*G)
      XSPP = -C1*XM(5)*SUM(1) - 0.5*C1*SUM(6) +C1*XM(5)*(XM(4)*SUM(3)-
     1   XM(3)*SUM(4))
      RETURN
      END
      FUNCTION XSPQ   ( XM, SUM, G, XMOT )
C
C
C     COMPUTE THE SHORT PERIODIC VARIATION OF Q.  SEE EQ. (42).
C
C
      DIMENSION SUM(6), XM(6)
      C1 = 1./(2.*XMOT*XMOT*XM(1)*XM(1)*G)
      XSPQ = -C1*XM(6)*SUM(1) + 0.5*C1*SUM(5) + C1*XM(6)*( XM(4)*SUM(3)-
     1   XM(3)*SUM(4))
      RETURN
      END
      FUNCTION XJLMP(L,M,P,I)
C
C
C     EVALUATE THE INCLINATION FUNCTION J.  SEE EQ. (52).
C
C
      REAL JLMP, I
      INTEGER F, ALPHA, AALPHA, P2
      C = COS(0.5*I)
      S = SIN(0.5*I)
      ALPHA = M-L+2*P
      MALPHA = -ALPHA
      AALPHA = IABS(ALPHA)
      L1 = L-M
      K = 0.5*L1
      DO  5    JJJ = 1,10,2
         IF (L1.EQ.JJJ) K = K + 1
    5 CONTINUE
      L2 = 2*L-2*P
      P2 = 2*P
      J1 = 0
      J2 = L1
      L3 = L+M
      L4 = L-P
      FMULT = (FACT(L3)/(FACT(P)*FACT(L4)*(2.**L)))*((-1.)**K)
      IF (MALPHA.GT.J1) J1 = MALPHA
      IF (L2.LT.J2) J2=L2
      IF (J1.GT.J2) PRINT 10, J1, J2
   10 FORMAT(1H0,20X,*PROGRAM TERMINATED IN FUNCTION  JLMP----J1 GT J2--
     A     J1 AND J2 =*,2I5)
      IF (J1.GT.J2) STOP
      JLMP = 0.00
      J1 = J1+1
```

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```
      J2 = J2+1
      DO 20   JJ = J1,J2
         J = JJ-1
         L5 = L1-J
         F = ((-1.)**J)*BINOM(L2,J)*BINOM(P2,L5)*FMULT
         JLMP = JLMP+F*(C**(2*L-ALPHA-2*J))*(S**(ALPHA-AALPHA+2*J))
   20 CONTINUE
      XJLMP = JLMP
      RETURN
      END
      FUNCTION DJLMP(L,M,P,I)
C
C
C     EVALUATE THE DERIVATIVE OF THE INCLINATION FUNCTION WITH RESPECT
C     TO  COS(.5*I) . SEE EQ. (68).
C
C
      REAL  I
      INTEGER P, ALPHA, AALPHA, P2
      C = COS(0.5*I)
      S = SIN(0.5*I)
      ALPHA = M-L+2*P
      MALPHA = -ALPHA
      AALPHA = IABS(ALPHA)
      L1 = L-M
      K = 0.5*L1
      DO  5    JJJ = 1,10,2
         IF (L1.EQ.JJJ) K = K + 1
    5 CONTINUE
      L2 = 2*L-2*P
      P2 = 2*P
      J1 = 0
      J2 = L1
      L3 = L+M
      L4 = L-P
      FMULT = (FACT(L3)/(FACT(P)*FACT(L4)*(2.**L)))*((-1.)**K)
      IF (MALPHA.GT.J1) J1 = MALPHA
      IF (L2.LT.J2) J2=L2
      IF (J1.GT.J2) PRINT 10, J1, J2
   10 FORMAT(1H0,20X,*PROGRAM TERMINATED IN FUNCTION DJLMP----J1 GT J2--
     A     J1 AND J2 =*,2I5)
      IF (J1.GT.J2) STOP
      DLMP = 0.0
      J1 = J1+1
      J2 = J2+1
      DO 20   JJ = J1,J2
         J = JJ-1
         L5 = L1-J
         F = ((-1.)**J)*BINOM(L2,J)*BINOM(P2,L5)*FMULT
         DLMP = DLMP + F*(C**(2*L-ALPHA-2*J-1))*(S**(ALPHA-AALPHA))*
     A     ((2*L-AALPHA)*(S**(2*J))-(2*J+ALPHA-AALPHA)*(S**(2*J-2)))
   20 CONTINUE
      DJLMP = DLMP
      RETURN
      END
```

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```
      FUNCTION XKLPQ(L,P,Q,GAMA)
C
C
C     EVALUATE THE ECCENTRICITY FUNCTION K.  SEE EQS. (53)-(54).
C
C
      INTEGER  P,Q,R,T,AQ,RR,TT,PP,PU,PUP
      ITST = 2*P - L
      XLPQ = 0.00
      IF (Q.EQ.ITST) GO TO 100
      L1 = -2*P
      L2 = 2*P - 2*L
      F1 = 0.5*(L - 2*P + Q)
      AQ = IABS(Q)
      F2 = 1. + GAMA
      F3 = 1. - GAMA
      F4 = ((-1.)**AQ)*(2.**L)*(F2**(-L-AQ))
      DO 10  KK = 1, 3
         K = KK-1
         KU = AQ + KK
            DO 20  RR = 1,KU
               R = RR - 1
                 DO 30  TT = 1,KK
                    T = TT-1
                    L3 = AQ + K - R
                    L4 = K - T
                    IF(Q.GE.0) BIFAC=((-1.)**R)*BINOM(L2,L3)*BINOM(L1,L4
     A                                                                  )
                    IF(Q.LT.0)BIFAC=((-1.)**T)*BINOM(L1,L3)*BINOM(L2,L4)
      TERM=F4*(BIFAC/(FACT(R)*FACT(T)))*(F1**(R+T))*(F2**(R+T-K))*(F3**K
     A                                                                  )
                    XLPQ = XLPQ + TERM
                    TST = ABS(TERM)
                    TST1 = 0.01 * XLPQ
                    TST1 = ABS ( TST1)
                    IF((TST.LT.TST1).AND.(TST.NE.0.00)) GO TO 40
   30            CONTINUE
   20         CONTINUE
   10      CONTINUE
      PRINT 11,  L,P,Q,GAMA,TST,TST1
   11 FORMAT (1H0,3X,*KLPQ DID NOT CONVERGE- L P Q GAMA TST TST1=*,3I5,
     A     3F15.7,//)
      GO TO 40
  100 IPP = IABS(ITST)
      L6 = L-1
      PP = (L - IPP)/2
      PU = PP - 1
      IF (PU.LT.0) GO TO 40
      PUP = PU + 1
      DO  50   KK = 1,PUP
         K = KK - 1
         L7 = 2*K + IPP
         XLPQ = XLPQ+(GAMA**(1.-2.*L))*BINOM(L6,L7)*BINOM(L7,K)*(2.**(-L
     A   7))*((1.-GAMA*GAMA)**K)
   50 CONTINUE
```

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```
   40 XKLPQ = XLPQ
      RETURN
      END
      FUNCTION DKLPQ(L,P,Q,GAMA)
C
C
C     EVALUATE THE DERIVATIVE OF THE ECCENTRICITY FUNCTION K WITH RESPECT
C     TO GAMMA  ( = SQRT(1. - E*E) )   . SEE EQS. (72)-(74).
C
C
      INTEGER  P,Q,R,T,AQ,RR,TT,PP,PU,PUP
      ITST = 2*P - L
      XLPQ = 0.00
      X1 = XKLPQ(L,P,Q,GAMA)
      IF (Q.EQ.ITST) GO TO 100
      L1 = -2*P
      L2 = 2*P - 2*L
      F1 = 0.5*(L - 2*P + Q)
      AQ = IABS(Q)
      F2 = 1. + GAMA
      F3 = 1. - GAMA
      F4 = ((-1.)**AQ)*(2.**L)*(F2**(-L-AQ))
      X1 = (( -L-AQ )/ F2)*X1
      DO 10  KK = 1, 3
         K = KK-1
         KU = AQ + KK
            DO 20  RR = 1,KU
               R = RR - 1
                  DO 30  TT = 1,KK
                     T = TT-1
                     L3 = AQ + K - R
                     L4 = K - T
                     IF(Q.GE.0) BIFAC= ((-1.)**R)*BINOM(L2,L3)*BINOM(L1,L4
     A                                                                     )
                     IF(Q.LT.0)BIFAC=((-1.)**T)*BINOM(L1,L3)*BINOM(L2,L4)
      TERM=F4*(BIFAC/(FACT(R)*FACT(T)))*(F1**(R+T))*(F2**(R+T-K-1))*
     A    ((F3**K)*(R+T-K)-K*F2*(F3**(K-1)))
                     XLPQ = XLPQ + TERM
                     TST = ABS(TERM)
                     TST1 = 0.01 * XLPQ
                     TST1 = ABS ( TST1)
                     IF((TST.LT.TST1).AND.(TST.NE.0.00)) GO TO 40
   30             CONTINUE
   20          CONTINUE
   10       CONTINUE
      PRINT 11,  L,P,Q,GAMA,TST,TST1
   11 FORMAT (1H0,3X,*KLPQ DID NOT CONVERGE- L P Q GAMA TST TST1=*,3I5,
     A    3F15.7,//)
      GO TO 40
  100 IPP = IABS(ITST)
      X1 = (( -2*L+1)/GAMA) * X1
      L6 = L-1
      PP = (L - IPP)/2
      PU = PP - 1
      IF (PU.LT.0) GO TO 40
```

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```
      PUP = PU + 1
      DO  50   KK = 1,PUP
         K = KK - 1
         L7 = 2*K + IPP
         XLPQ = XLPQ-2.*(GAMA**(2.-2.*L))*BINOM(L6,L7)*BINOM(L7,K)*
     A         (2.**(-L7))*K*((1.-GAMA*GAMA)**(K-1))
   50 CONTINUE
   40 DKLPQ = XLPQ + X1
      RETURN
      END
      FUNCTION RLMPQ(L,M,P,Q,A,B,C,D)
C
C
C     EVALUATE THE R FUNCTION. SEE EQ. (55).
C
C
      INTEGER ALPHA,AALPHA,P,Q,AQ,U1,U2,U,UU1,UU2,UU
      ALPHA = M - L + 2*P
      AALPHA =IABS(ALPHA)
      AQ = IABS(Q)
      R = 0.00
      K = 0.5*(AQ + AALPHA)
      KK = K + 1
      DEL = 1.0
      IPRD = Q*ALPHA
      DO 10  NN = 1, KK
         N = NN - 1
         L 1 = 2*N - AALPHA
         L2= 2*N
         U1 = 0
         IF (L1.GT. 0) U1 = L1
         U2 = L2
         IF ( AQ .LT. L2) U2 = AQ
         UU1 = U1 + 1
         UU2 = U2 + 1
         IF(U1.GT.U2) PRINT 11, U1, U2
   11 FORMAT (1H0,3X,*LOWER BOUND GT UPPER IN SUM OVER U IN FUNCTION RLM
     APQ - U1  U2 =*,2I6,//)
         IF (U1.GT.U2) STOP
               DO 20   UU = UU1, UU2
               U = UU - 1
               IF (IPRD.LT.0) DEL = (-1.0)**U
               N1 = 2*N - U
            R = R + ((-1.0)**(N+U))*DEL*BINOM(AQ,U)*BINOM(AALPHA,N1)*
     A      (A**(AQ-U))* (B**U)*(C**(AALFHA-2*N+U))*(D**(2*N-U))
   20          CONTINUE
   10    CONTINUE
      RLMPQ = R
      RETURN
      END
      FUNCTION BILMPQ(L,M,P,Q,A,B,C,D)
C
C
C     EVALUATE THE I FUNCTION.  SEE EQ. (56).
C
```

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```
C
      INTEGER ALPHA,AALPHA,P,Q,AQ,U1,U2,U,UU1,UU2,UU
      ALPHA = M - L + 2*P
      AALPHA =IABS(ALPHA)
      AQ = IABS(Q)
      R = 0.00
      K = 0.5*(AQ + AALPHA - 1)
      KK = K + 1
      DEL = 1.0
      IPRO = Q*ALPHA
      DO 10  NN = 1, KK
         N = NN - 1
         L 1 = 2*N - AALPHA + 1
         L2= 2*N + 1
         U1 = 0
         IF (L1.GT. 0) U1 = L1
         U2 = L2
         IF ( AQ .LT. L2) U2 = AQ
         UU1 = U1 + 1
         UU2 = U2 + 1
         IF ( U1 .GT. U2)  R = 0.0
         IF ( U1 .GT. U2)  GO TO 30
               DO 20   UU = UU1, UU2
               U = UU - 1
               IF (IPRO.LT.0) DEL = (-1.0)**U
      IF (( IPRO.EQ.0).AND.(Q.LT.0)) DEL=(-1.0)**U
       IF((IPRO.EQ.0).AND.(ALPHA.LT.0)) DEL = (-1.0)**U
               N1 = 2*N - U + 1
            R = R+((-1.0)**(N+U+1))*DEL*BINOM(AQ,U)*BINOM(AALPHA,N1)*
     A          (A**(AQ-U))*(B**U)*(C**(AALPHA-2*N+U-1))*(D**(2*N-U+1))
   20          CONTINUE
   10    CONTINUE
   30 CONTINUE
      BILMPQ = R
      RETURN
      END
      FUNCTION BINOM(M,N)
C
C
C     EVALUATE  A BINOMIAL EXPANSION COEFFICIENT.
C
C
      IF (N.LT.0) BINOM=0.00
      IF(N.LT.0) RETURN
      IF(N.EQ.0) BINOM = 1.00
      IF (N.EQ.0) RETURN
      J = M
      IF (M.LT.0 )  M= N-M-1
      L = M - N
      IF ( L . LT . 0 )   BINOM = 0.00
      IF ( L . LT . 0 )   RETURN
      BINOM = FACT(M) / ( FACT(N)*FACT(L))
      IF (J.LT.0) BINOM = ((-1.0)**N) * BINOM
      M = J
      RETURN
```

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```
      END
      FUNCTION FACT(K)
C
C
C     EVALUATE A FACTORIAL.
C
C
      IF (K.LT.0) PRINT 10,K
   10 FORMAT(1H0,20X,*PROGRAM HAS TERMINATED DUE TO FACTORIAL OF A NEGAT
     AIVE INTEGER---K=*,I5)
      IF (K.LT.0) STOP
      FACT = 1.0
      IF (K.EQ.0) RETURN
      DO  20  I = 1,K
         FACT = FACT * I
   20 CONT.NUE
      RETURN
      END
6/7/8/9
```

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